Solutions for Tutorial 7: Rewrite Rules and Induction

Exercise 1

Recall from the lectures that for rewrite rules $L1 \Rightarrow R_1$ and $L2 \Rightarrow R_2$, a critical pair can be defined as:

$$\langle R_1[\theta], L_1[\theta]/\{R_2[\theta]/s\}\rangle$$

where $\theta = mgu$ of s (subpart of L_1) and L_2.

Now if we take:

$$\overbrace{(\underbrace{u+v}_{s}) + w}^{L_1} \Rightarrow \overbrace{u + (v + w)}^{R_1}$$

$$\overbrace{s(x) + y}^{L_2} \Rightarrow \overbrace{x + s(y)}^{R_2}$$

Then $\theta = [s(x)/u, y/v]$, so the critical pair is given by

$$\langle\ s(x) + (y + w), (x + s(y)) + w\rangle$$

More concisely: The expression $s(x) + y$ unifies with $u + v$ with common instance $s(x) + y$. $(s(x) + y) + w$ can be rewritten to either $(x + s(y)) + w$ or $s(x) + (y + w)$. So the critical pair is:

$$\langle\ s(x) + (y + w), (x + s(y)) + w\rangle$$

Exercise 2

Two examples of critical pairs that are not joinable are: 1) starting from $(X \cdot 0) \cdot Z$ we can get the critical pair $\langle X \cdot (0 \cdot Z), X \cdot Z\rangle$; and 2) starting from $(X \cdot i(X)) \cdot Z$ we can get the critical pair $\langle 0 \cdot Z, X \cdot (i(X) \cdot Z)\rangle$. (2 marks for each critical pair). To get full marks, must mention that the critical pair is not joinable (or "conflatable")

Exercise 3

1. An appropriate induction rule is:

$$\frac{\Gamma \vdash P(\texttt{LEAF}\ x) \quad \Gamma, P(x_1), P(x_2) \vdash P(\texttt{NODE}\ a\ x_1\ x_2)}{\Gamma \vdash \forall t. P(t)}$$

2. An Isabelle definition (see theory file for proof):

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primrec MIRROR :: "'a TREE => 'a TREE" where
  "MIRROR (LEAF x) = LEAF x"
| "MIRROR (NODE x l r) = NODE x (MIRROR r) (MIRROR l)"
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