Exercise 1 (Past exam question)
Find a non-trivial critical pair of the following pair of rewrite rules:

\[ s(x) + y \Rightarrow x + s(y) \] and \[ (u + v) + w \Rightarrow u + (v + w) \]

Explain your reasoning.

Exercise 2 (Past exam question)
Consider the following set of rewrite rules:

\[
\begin{align*}
(X \cdot Y) \cdot Z & \Rightarrow X \cdot (Y \cdot Z) \\
X \cdot 0 & \Rightarrow X \\
X \cdot i(X) & \Rightarrow 0
\end{align*}
\]

Give two ways in which this system of rewrite rules is not locally confluent. Explain your answer in terms of critical pairs.

Exercise 3
Consider the following (Isabelle) datatype definition:

\[
\textbf{datatype 'a TREE} = \text{LEAF 'a} \mid \text{NODE '}a 'a \text{ TREE} 'a \text{ TREE}
\]

1. Give an induction rule appropriate for proofs by (structural) induction involving the \text{TREE} datatype.

2. Define a function \textsc{Mirror} that recursively flips the nodes in the left and right subtrees of a tree as defined above.

3. Formalize your definition of \textsc{Mirror} in Isabelle and give a structured (Isar) proof that \textsc{Mirror(Mirror t)} = t.