Automated Reasoning: Tutorial 7
Rewrite Rules and Induction

Exercise 1 (Past exam question)

Find a non-trivial critical pair of the following pair of rewrite rules:

$$s(x) + y \Rightarrow x + s(y) \text{ and } (u + v) + w \Rightarrow u + (v + w)$$

Explain your reasoning.

Exercise 2 (Past exam question)

Consider the following set of rewrite rules:

$$\begin{array}{lcl} (X \cdot Y) \cdot Z & \Rightarrow & X \cdot (Y \cdot Z) \\ X \cdot 0 & \Rightarrow & X \\ X \cdot i(X) & \Rightarrow & 0 \end{array}$$

Give two ways in which this system of rewrite rules is not locally confluent. Explain your answer in terms of critical pairs.

Exercise 3

Consider the following (Isabelle) datatype definition:

```
datatype 'a TREE = LEAF 'a | NODE 'a "'a TREE" "'a TREE"
```

1. Give an induction rule appropriate for proofs by (structural) induction involving the `TREE` datatype.
2. Define a function `MIRROR` that recursively flips the nodes in the left and right subtrees of a tree as defined above.
3. Formalize your definition of `MIRROR` in Isabelle and give a structured (Isar) proof that `MIRROR`(`MIRROR` t) $= t$.