Solutions for Tutorial 6:
Unification and Rewrite Rules

Exercise 1

(a) 1. \((X \equiv 2) \land (X \equiv 2)\)  \hspace{1cm} \text{(by decompose)}
2. \((2 \equiv 2) \land (X \equiv 2)\)  \hspace{1cm} \text{(by eliminate)}
3. \(X \equiv 2\)  \hspace{1cm} \text{(by delete)}

Succeeds with \(X=2\)

(b) 1. \((X \equiv 2 + 2) \land (X \equiv 4)\)  \hspace{1cm} \text{(by decompose)}
2. \((4 \equiv 2 + 2) \land (X \equiv 4)\)  \hspace{1cm} \text{(by eliminate)}

Fail. There is a conflict matching 4 and 2 + 2.

(c) 1. \((X \equiv a) \land (Y \equiv g(b)) \land (Y \equiv g(b))\)  \hspace{1cm} \text{(by decompose)}
2. \((X \equiv a) \land (g(b) \equiv g(b)) \land (Y \equiv g(b))\)  \hspace{1cm} \text{(by eliminate)}
3. \((X \equiv a) \land (Y \equiv g(b))\)  \hspace{1cm} \text{(by delete)}

Succeeds with \(X=a\) and \(Y = g(b)\)

(d) 1. \((X \equiv a) \land (b \equiv Y)\)  \hspace{1cm} \text{(by decompose)}

Fail as target contains a variable.

Exercise 2

(a) 1. \((X \equiv a) \land (b \equiv Y)\)  \hspace{1cm} \text{(by decompose)}
2. \((X \equiv a) \land (Y \equiv b)\)  \hspace{1cm} \text{(by switch)}

Succeeds with \(X=a\) and \(Y = b\)

(b) 1. \((X \equiv Y) \land (b \equiv a)\)  \hspace{1cm} \text{(by decompose)}

Fails \text{(by conflict).}
(c) 1. \( (X \equiv f(Y)) \land (a \equiv Y) \) (by decompose)
2. \( (X \equiv f(Y)) \land (Y \equiv a) \) (by switch)
3. \( (X \equiv f(a)) \land (Y \equiv a) \) (by eliminate)

Succeeds with \( X \equiv f(a) \) and \( Y \equiv a \).

(d) 1. \( (X \equiv f(Y)) \land (g(X) \equiv Y) \) (by decompose)
2. \( (X \equiv f(Y)) \land (g(f(Y)) \equiv Y) \) (by eliminate)
3. \( (X \equiv f(Y)) \land (Y \equiv g(f(Y))) \) (by switch)

Fails due to occurs check.

(e) 1. \( (a + X \equiv a) \land (b \equiv Y) \) (by decompose)
2. \( (a \equiv a + X) \land (b \equiv Y) \) (by switch)

Fails due to occurs check.

**Exercise 3**

A suitable property is that \( g(X, X) = X \), for all \( X \). Adding this to the unification algorithm means that the two terms given can unify with the substitution \( X = f(a, a) \) and \( Y = a \). This can be shown by performing the substitutions on both terms and applying the property of \( g \).

**Exercise 4**

One normal form is:

\[
\neg ( \neg p \land (q \lor \neg r)) \\
= \neg \neg p \lor \neg (q \lor \neg r) \quad \text{(From rule 2)} \\
= p \lor \neg (q \lor \neg r) \quad \text{(From rule 1)} \\
= p \lor (\neg q \land \neg \neg r) \quad \text{(From rule 3)} \\
= p \lor (\neg q \land r) \quad \text{(From rule 1)}
\]
Exercise 5

To show that the rule terminates we need some decreasing measure. Could choose:

- Average depth of parse tree decreases
- Number of arithmetic operations decreases
- Number of terms decreases