Solutions for Tutorial 6: Unification and Rewrite Rules

Exercise 1

(a) 1. $(X \equiv 2) \wedge (X \equiv 2)$ (by *decompose*)
2. $(2 \equiv 2) \wedge (X \equiv 2)$ (by *eliminate*)
3. $X \equiv 2$ (by *delete*)

Succeeds with X=2

(b) 1. $(X \equiv 2+2) \wedge (X \equiv 4)$ (by *decompose*)
2. $(4 \equiv 2+2) \wedge (X \equiv 4)$ (by *eliminate*)

Fail. There is a *conflict* matching 4 and $2+2$.

(c) 1. $(X \equiv a) \wedge (Y \equiv g(b)) \wedge (Y \equiv g(b))$ (by *decompose*)
2. $(X \equiv a) \wedge (g(b) \equiv g(b)) \wedge (Y \equiv g(b))$ (by *eliminate*)
3. $(X \equiv a) \wedge (Y \equiv g(b))$ (by *delete*)

Succeeds with X=a and $Y = g(b)$

(d) 1. $(X \equiv a) \wedge (b \equiv Y)$ (by *decompose*)

Fail as target contains a variable.

Exercise 2

(a) 1. $(X \equiv a) \wedge (b \equiv Y)$ (by *decompose*)
2. $(X \equiv a) \wedge (Y \equiv b)$ (by *switch*)

Succeeds with X=a and $Y = b$

(b) 1. $(X \equiv Y) \wedge (b \equiv a)$ (by *decompose*)

Fails (by *conflict*).

(c) 1. $(X \equiv f(Y)) \wedge (a \equiv Y)$ (by *decompose*)
2. $(X \equiv f(Y)) \wedge (Y \equiv a)$ (by *switch*)
3. $(X \equiv f(a)) \wedge (Y \equiv a)$ (by *eliminate*)

Succeeds with $X \equiv f(a)$ and $Y \equiv a$.

(d) 1. $(X \equiv f(Y)) \wedge (g(X) \equiv Y)$ (by *decompose*)
2. $(X \equiv f(Y)) \wedge (g(f(Y)) \equiv Y)$ (by *eliminate*)
3. $(X \equiv f(Y)) \wedge (Y \equiv g(f(Y)))$ (by *switch*)

Fails due to occurs check.

(e) 1. $(a + X \equiv a) \wedge (b \equiv Y)$ (by *decompose*)
2. $(a \equiv a + X) \wedge (b \equiv Y)$ (by *switch*)

Fails due to occurs check.

Exercise 3

A suitable property is that $g(X, X) = X$, for all X. Adding this to the unification algorithm means that the two terms given can unify with the substitition $X = f(a, a)$ and $Y = a$. This can be shown by performing the substitutions on both terms and applying the property of g

Exercise 4

One normal form is:

$\neg\ (\ \neg p \wedge (q \vee \neg r))$
$= \neg\neg p \vee \neg(q \vee \neg r))$ (From rule 2)
$= p \vee \neg(q \vee \neg r))$ (From rule 1)
$= p \vee (\neg q \wedge \neg\neg r))$ (From rule 3)
$= p \vee (\neg q \wedge r))$ (From rule 1)

Exercise 5

To show that the rule terminates we need some decreasing measure. Could choose:

- Average depth of parse tree decreases
- Number of arithmetic operations decreases
- Number of terms decreases