Solutions for Tutorial 5: Unification and Rewrite Rules

**Exercise 1**

(a) 1. \((X ≡ 2) ∧ (X ≡ 2)\) (by decompose)
2. \((2 ≡ 2) ∧ (X ≡ 2)\) (by eliminate)
3. \(X ≡ 2\) (by delete)

Succeeds with \(X ≡ 2\)

(b) 1. \((X ≡ 2 + 2) ∧ (X ≡ 4)\) (by decompose)
2. \((4 ≡ 2 + 2) ∧ (X ≡ 4)\) (by eliminate)

Fails due to conflict

(c) 1. \((X ≡ a) ∧ (Y ≡ g(b)) ∧ (Y ≡ g(b))\) (by decompose)
2. \((X ≡ a) ∧ (g(b) ≡ g(b)) ∧ (Y ≡ g(b))\) (by eliminate)
3. \((X ≡ a) ∧ (Y ≡ g(b))\) (by delete)

Succeeds with \(X ≡ a\) and \(Y ≡ g(b)\)

(d) 1. \((X ≡ a) ∧ (b ≡ Y)\) (by decompose)

Fails as target contains a variable.

**Exercise 2**

(a) 1. \((X ≡ a) ∧ (b ≡ Y)\) (by decompose)
2. \((X ≡ a) ∧ (Y ≡ b)\) (by switch)

Succeeds with \(X ≡ a\) and \(Y ≡ b\)

(b) 1. \((X ≡ Y) ∧ (b ≡ a)\) (by decompose)

Fails due to conflict
(c) 1. \((X \equiv f(Y)) \land (a \equiv Y)\) (by decompose)
2. \((X \equiv f(Y)) \land (Y \equiv a)\) (by switch)
3. \((X \equiv f(a)) \land (Y \equiv a)\) (by eliminate)

Succeeds with \(X \equiv f(a)\) and \(Y \equiv a\).

(d) 1. \((X \equiv f(Y)) \land (g(X) \equiv Y)\) (by decompose)
2. \((X \equiv f(Y)) \land (g(f(Y)) \equiv Y)\) (by eliminate)
3. \((X \equiv f(Y)) \land (Y \equiv g(f(Y)))\) (by switch)

Fails due to occurs check.

(e) 1. \((a + X \equiv a) \land (b \equiv Y)\) (by decompose)

Fails due conflict.

**Exercise 3**

A suitable property is that \(g(X,Y) = X\), for all \(X\). Adding this to the unification algorithm means that the two terms given can unify with the substitution \(X = f(a,a)\) and \(Y = a\). This can be shown by performing the substitutions on both terms and applying the property of \(g\).

**Exercise 4**

One normal form is:

\[
\neg \left( \neg p \land (q \lor \neg r) \right) \\
= \neg \neg p \lor \neg(q \lor \neg r) \quad \text{(From rule 2)} \\
= p \lor \neg(q \lor \neg r) \quad \text{(From rule 1)} \\
= p \lor (\neg q \land \neg \neg r) \quad \text{(From rule 3)} \\
= p \lor (\neg q \land r) \quad \text{(From rule 1)}
\]
Exercise 5

To show that the rule terminates we need some decreasing measure. We could choose (among other possibilities):

- Number of arithmetic operations decreases.
- Number of terms decreases.