Introduction

In the following exercise, you will formally verify the correctness of some simple programs using Isabelle’s Hoare Logic library. This library allows you to formalise the specifications of programs of a simple programming language in the form of Hoare triples.

The supported programming language includes the following constructs:

- Local variable declaration: VARS x y z
- Sequence: p ; q
- Skip (do nothing): SKIP
- Variable assignment: x := 0
- Conditional: IF cond THEN p ELSE q FI
- Loop: WHILE cond INV {invariant} DO p OD

A program X with precondition P and postcondition Q can be specified as the Hoare triple and, when mechanized in Isabelle, must begin with a local variable declaration VARS including at least one local variable, i.e. a Hoare triple is specified in Isabelle as follows:

"VARS a {P} X {Q}"

Note that a loop invariant must be explicitly specified for each while loop using the INV operator.

The automatic Isabelle tactic vcg, can be used to extract verification conditions from the Hoare triples and convert them to Isabelle subgoals. The tactic vcg_simp combines the capabilities of vcg with simplification.

Use the provided Isabelle theory file to carry out the exercise. Note that you can use any pre-defined Isabelle type or function (from the library) in the program specifications.

Exercise

Verify the correctness of the following programs. Some of the examples require that you introduce the appropriate invariant Inv.

a). The minimum of two integers x and y:

lemma Min: "VARS (z :: int) {True}
IF x ≤ y THEN z := x ELSE z := y FI
{ z = min x y }"
b). Iteratively copy an integer variable $x$ to $y$:

lemma Copy: "VARS (a :: int) y
{0 ≤ x}
  a := x; y := 0;
  WHILE a ≠ 0
  INV { Inv }
  DO y := y + 1 ; a := a - 1 OD
{ x = y }"
-- "Replace Inv with your invariant."

c). Iterative multiplication through addition:

lemma Multi1: "VARS (a :: int) z
{0 ≤ y}
  a := 0; z := 0;
  WHILE a ≠ y
  INV { Inv }
  DO
    z := z + x ;
    a := a + 1
  OD
{ z = x * y }"
-- "Replace Inv with your invariant."

d). Alternative multiplication algorithm:

lemma Multi2: "VARS (z :: int) y
{ y = y0 ∧ 0 ≤ y}
  z := 0;
  WHILE y ≠ 0
  INV { Inv }
  DO
    z := z + x;
    y := y - 1
  OD
{ z = x * y0 }"
-- "Replace Inv with your invariant."

e). A factorial algorithm:

lemma DownFact: "VARS (z :: nat) (y::nat)
{True}
  z := x; y := 1;
  WHILE z > 0
  INV { Inv }
  DO
    y := y * z ;
    z := z - 1
  OD
y = fact x"
f). Integer division of $x$ by $y$:

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lemma Div: "VARS (r :: int) d
{\text{y} \neq 0}
  r := x; d := 0;
  WHILE \text{y} \leq r
  \text{INV} \{ \text{Inv} \}
  DO
  r := r - y;
  d := d + 1
  OD
{ \text{Postcondition} }"
```

-- "Replace Inv with your invariant."

-- "Replace Postcondition with an appropriate postcondition that reflects
the expected behaviour of the algorithm."