

Automated Reasoning: Tutorial 4

Exercise 1

Provide structured (declarative) proofs the following statements in Isabelle/Isar:

- a). $(P \longrightarrow (Q \longrightarrow R)) \longrightarrow ((P \longrightarrow Q) \longrightarrow (P \longrightarrow R))$
- b). $(\forall x. P x \longrightarrow Q) \longrightarrow (\exists x. P x \longrightarrow Q)$
- c). $\forall x. \neg P x$, assuming that $\neg \exists x. P x$ is true
- d). $\exists x. \neg P x$, assuming that $\neg \forall x. P x$ is true
- e). $(R \longrightarrow P) \longrightarrow (((\neg R \vee P) \longrightarrow (Q \longrightarrow S)) \longrightarrow (Q \longrightarrow S))$

Exercise 2

Provide a structured proof of the following theorem (from the additional Isabelle exercises) using Isabelle/Isar. Use case distinctions and/or proof by contradiction.

$$(\forall x. \neg rich\ x \longrightarrow rich\ (father\ x)) \longrightarrow \exists x. rich\ (father\ (father\ x)) \wedge rich\ x$$