Automated Reasoning: Solutions for Tutorial 2

Exercise 1

An interpretation with a domain containing a single member (i.e. a singleton set) will satisfy the formula. For example, if the domain is $\{a\}$ then either $p(a)$ is true or $p(a)$ is false. In either case, the statement is true.

Note: There are many other possible solutions.

Exercise 2

See the Isabelle theory file.

Note: The statements from this exercise can be proved in several ways. Moreover, Isabelle has pre-proved lemmas that can make some of these proofs fairly trivial. You should attempt to prove the statements using only the basic Natural Deduction rules for classical FOL.

Exercise 3

1. $(\forall x. P\, x \to Q) \to (\exists x. P\, x \to Q)$

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\cfrac{\overline{P\, a \to Q \vdash P\, a \to Q}\; assumption}
{\forall x. P\, x \to Q \vdash P\, a \to Q}\; allE}
{\forall x. P\, x \to Q \vdash \exists x. P\, x \to Q}\; exI}
{\vdash (\forall x. P\, x \to Q) \to (\exists x. P\, x \to Q)}\; allI
$$

2. $\forall x. \neg P\, x$, assuming that $\neg \exists x. P\, x$

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\cfrac{\overline{P\, x_0 \vdash P\, x_0}\; assumption}
{P\, x_0 \vdash \exists x. P\, x}\; exI}
{\neg \exists x. P\, x, P\, x_0 \vdash \bot}\; notE}
{\neg \exists x. P\, x \vdash \neg P\, x_0}\; notI}
{\neg \exists x. P\, x \vdash \forall x. \neg P\, x}\; allI
$$

3. $\exists x.\neg P\,x$, assuming that $\neg\forall x.P\,x$ is true

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