Automated Reasoning: Solutions to Tutorial Exercise 1

Exercise 1

1. Cats chase mice or birds, but not at the same time.
   This can be represented as: $(M \vee B) \wedge \neg(M \wedge B)$
   where M: Cats chase mice B: Cats chase birds

M	B	$M \vee B$	$\neg(M \wedge B)$	$(M \vee B) \wedge \neg(M \wedge B)$
t	t	t	f	f
t	f	t	t	t
f	t	t	t	t
f	f	f	t	f

2. If it rains the beach will be empty.
   This can be represented as: $R \longrightarrow E$
   where R: It rains E: Beach is empty

R	E	$R \longrightarrow E$
t	t	t
t	f	f
f	t	t
f	f	t

3. If Jane bought a piano today, she either sold her old one or took out a bank loan.
   This can be represented as: $P \longrightarrow S \vee B$
   where P: Jane bought a piano today S: Jane sold her old piano
   B: Jane took out a bank loan

P	S	L	$P \longrightarrow S \vee B$
t	t	t	t
t	t	f	t
t	f	t	t
t	f	f	f
f	t	t	t
f	t	f	t
f	f	t	t
f	f	f	t

Exercise 2

The proposition $P \wedge (P \longrightarrow Q)$ is satisfiable if there is some interpretation which evaluates to *true*. It is valid if all interpretations evaluate to *true*.

$P \wedge (P \longrightarrow Q)$ is satisfiable since it evaluates to *true* when P is *true* and Q is *true*..

$P \wedge (P \longrightarrow Q)$ is not valid since it evaluates to *false* when P is *false*.

Exercise 3

Connective	Expression using |
$\neg$	$p \mid$ p
$\wedge$	$(\ p \mid q\ ) \mid (p \mid q)$
$\vee$	$(p \mid p\ ) \mid (\ q \mid q\ )$
$\longrightarrow$	$p \mid (q \mid q)$

Some notes:

- $p \wedge q$ is the same as $\neg\ (p \mid q)$
- $p \vee q$ is the same as $\neg(\neg\ p \wedge \neg\ q)$
- $p \longrightarrow q$ is the same as $\neg\ p \vee q$

Exercise 4

One possible ND proof:

$$\dfrac{[Q]_3 \qquad \dfrac{\dfrac{\dfrac{}{R \vee \neg R}\ \textit{excluded\_middle} \qquad \dfrac{\dfrac{[R]_4 \quad [R \to P]_1}{P}\ \textit{mp}}{\neg R \vee P}\ \textit{disjI2} \qquad \dfrac{[\neg R]_4}{\neg R \vee P}\ \textit{disjI1}}{\neg R \vee P}\ \textit{disjE}_4 \qquad [(\neg R \vee P) \to (Q \to S)]_2}{Q \to S}\ \textit{mp}}{\dfrac{\dfrac{\dfrac{S}{Q \to S}\ \textit{impI}_3}{((\neg R \vee P) \to (Q \to S)) \to (Q \to S)}\ \textit{impI}_2}{(R \to P) \to (((\neg R \vee P) \to (Q \to S)) \to (Q \to S))}\ \textit{impI}_1}\ \textit{mp}$$

Alternatively, since the above proof does an application of impI that can be omitted to give a more succinct derivation:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{R \vee \neg R}\ \textit{excluded\_middle} \qquad \dfrac{\dfrac{[R]_3 \quad [R \to P]_1}{P}\ \textit{mp}}{\neg R \vee P}\ \textit{disjI2} \qquad \dfrac{[\neg R]_3}{\neg R \vee P}\ \textit{disjI1}}{\neg R \vee P}\ \textit{disjE}_3 \qquad [(\neg R \vee P) \to (Q \to S)]_2}{Q \to S}\ \textit{mp}}{((\neg R \vee P) \to (Q \to S)) \to (Q \to S)}\ \textit{impI}_2}{(R \to P) \to (((\neg R \vee P) \to (Q \to S)) \to (Q \to S))}\ \textit{impI}_1$$

Note: The Isabelle theory file associated with this tutorial gives yet another proof that does not use the excluded middle axiom and Cut rule. It uses Isabelle's `ccontr` rule, which (as indicated in the lectures) is an alternative to excluded middle when it comes to making the logic classical.