Automated Reasoning: Tutorial 1

Exercise 1

Represent the following sentences in propositional logic, using the connectives \(\neg\), \(\rightarrow\), \(\land\) and \(\lor\) (make clear what your propositional variables stand for):

1. Cats chase mice or birds, but not at the same time.
2. If it rains the beach will be empty.
3. If Jane bought a piano today, she either sold her old one or took out a bank loan.

Also draw the truth tables for each statement.

Exercise 2

Is the proposition \(P \land (P \rightarrow Q)\) satisfiable? If so, give an interpretation that satisfies it. Is it valid? Why or why not?

Exercise 3

The truth table for the following NAND expression, \(p \mid q\) is:

\[
\begin{array}{c|c|c|c|c|c|c|c}
 p & q & p \mid q \\
 t & t & f \\
 t & f & t \\
 f & t & t \\
 f & f & t \\
\end{array}
\]

Show that \(\mid\) alone can be used to define the connectives: \(\neg\), \(\land\), \(\lor\) and \(\rightarrow\).

Exercise 4

Using natural deduction, give a (tree representation) proof the theorem:

\[(R \rightarrow P) \rightarrow (((\neg R \lor P) \rightarrow (Q \rightarrow S)) \rightarrow (Q \rightarrow S))\]
Exercise 5

Prove the following propositional statements in Isabelle:

1. \((P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R))\)
2. \(\neg\neg P \rightarrow P\)
3. \((P \rightarrow Q \land R) \rightarrow ((P \rightarrow Q) \land (P \rightarrow R))\)
4. \((\neg P \rightarrow Q) \rightarrow (\neg Q \rightarrow P)\)
5. \(P \lor \neg P\)

Exercise 6

Give tree representation proofs for the statements in the previous exercise annotating your steps using the Isabelle names of rules (e.g. conjI, impI, etc).