Recap

- Previously:
  - Reasoning by applying deduction rules

- This time: Introduction to Model Checking
  - Specifications as Formulas, Programs as Models
  - Programs are abstracted as Finite State Machines
  - Formulas are in Temporal Logic
Interpretation $\models$ Formula

The relationship between interpretations $M$ and formulas $P$:

$$M \models P$$

We say $M$ models $P$.

Questions we can ask:
**Interpretation $\models$ Formula**

The relationship between interpretations $M$ and formulas $P$:

$$M \models P$$

We say $M$ models $P$.

Questions we can ask:

1. For a fixed $P$, is $M \models P$ true for all $M$?
   - Validity of $P$
   - Did this via natural deduction rules in Isabelle

2. For a fixed $P$, is $M \models P$ true for some $M$?

3. For a fixed (class of) $M$, what $P$s make $M \models P$ true?
   - “Theory discovery”/“Learning from Data”/“Generalisation”
   - Not in this course :(

4. For a fixed $M$ and $P$, is it the case that $M \models P$?
   - Model Checking
Interpretation $\models$ Formula

The relationship between interpretations $M$ and formulas $P$:

$$M \models P$$

We say $M$ models $P$.

Questions we can ask:

1. For a fixed $P$, is $M \models P$ true for all $M$?
   - Validity of $P$
   - Did this via natural deduction rules in Isabelle

2. For a fixed $P$, is $M \models P$ true for some $M$?
   - Satisfiability
**Interpretation $\models$ Formula**

The relationship between interpretations $M$ and formulas $P$:

$$M \models P$$

We say $M$ models $P$.

Questions we can ask:

1. For a fixed $P$, is $M \models P$ true for all $M$?
   - Validity of $P$
   - Did this via natural deduction rules in Isabelle

2. For a fixed $P$, is $M \models P$ true for some $M$?
   - Satisfiability

3. For a fixed (class of) $M$, what $Ps$ make $M \models P$ true?
   - “Theory discovery”/“Learning from Data”/“Generalisation”
   - Not in this course : (
Interpretation $\models$ Formula

The relationship between interpretations $M$ and formulas $P$:

$$M \models P$$

We say $M$ models $P$.

Questions we can ask:

1. For a fixed $P$, is $M \models P$ true for all $M$?
   - Validity of $P$
   - Did this via natural deduction rules in Isabelle

2. For a fixed $P$, is $M \models P$ true for some $M$?
   - Satisfiability

3. For a fixed (class of) $M$, what $P$s make $M \models P$ true?
   - “Theory discovery”/“Learning from Data”/“Generalisation”
   - Not in this course :(

4. For a fixed $M$ and $P$, is it the case that $M \models P$?
   - Model Checking
Model Checking

At a high level, many tasks can be rephrased as model checking.

<table>
<thead>
<tr>
<th>“Interpretations” $M$</th>
<th>“Formulas” $P$</th>
<th>Task</th>
</tr>
</thead>
<tbody>
<tr>
<td>sequences of tokens</td>
<td>grammars</td>
<td>parsing</td>
</tr>
<tr>
<td>database tables</td>
<td>SQL queries</td>
<td>query execution</td>
</tr>
<tr>
<td>email texts</td>
<td>spam rules</td>
<td>spam detection</td>
</tr>
<tr>
<td>sequences of letters</td>
<td>dictionary</td>
<td>spellchecking</td>
</tr>
<tr>
<td>audio data</td>
<td>acoustic/lang. model</td>
<td>speech recognition</td>
</tr>
<tr>
<td>finite state machines</td>
<td>temporal logic</td>
<td>specification checking</td>
</tr>
</tbody>
</table>

Details differ widely, but question of “is this data consistent with this statement? (and to what degree?)” is extremely common.

Historically, “Model Checking” usually refers to the last one. This is the one we will cover over the next few lectures.
Model Checking has been used to:

- Check Microsoft Windows device drivers for bugs
  - The “Static Driver Verifier” tool
- The SPIN tool (http://spinroot.com):
  - Flood control barrier control software
  - Call processing software at Lucent
  - Parts of Mars Science Laboratory, Deep Space 1, Cassini, the Mars Exploration Rovers, Deep Impact
- PEPA (Performance Evaluation Process Algebra) http://www.dcs.ed.ac.uk/pepa/
  - Multiprocessor systems
  - Biological systems
Model Checking – Models

A model of some system has:

▶ A finite set of states
▶ A subset of states considered as the initial states
▶ A transition relation which, given a state, describes all states that can be reached “in one time step”.

Good for

▶ Software, sequential and concurrent
▶ Digital hardware
▶ Communication protocols

Refinements of this setup can handle: Infinite state spaces, Continuous state spaces, Continuous time, Probabilistic Transitions. Good for hybrid (i.e., discrete and continuous) and control systems.
Models are *always* abstractions of reality.

The Cure's *Pictures of You*:
I've been looking so long at these pictures of you
That I almost believe that they're real
I've been living so long with my pictures of you
That I almost believe that the pictures are
All I can feel
Do not do this: the pictures are not real.
Model Checking – Models

Models are *always* abstractions of reality.

- We must choose what to model and what not to model

In the words of The Cure’s ‘Pictures of You’:

*I’ve been looking so long at these pictures of you
That I almost believe that they’re real
I’ve been living so long with my pictures of you
That I almost believe that the pictures are
All I can feel

Do not do this: the pictures are not real.*
Models are *always* abstractions of reality.

- We must choose what to model and what not to model
- There will limitations forced by the formalism
  - *e.g.*, here we are limited to **finite state** models

---

In the words of The Cure's *Pictures of You*:

> I've been looking so long at these pictures of you
> That I almost believe that they're real
> I've been living so long with my pictures of you
> That I almost believe that the pictures are

Do not do this: the pictures are not real.
Models are *always* abstractions of reality.

- We must choose what to model and what not to model
- There will limitations forced by the formalism
  - *e.g.*, here we are limited to **finite state** models
- There will be things we do not understand sufficiently to model
  - *e.g.*, people
Model Checking – Models

Models are always abstractions of reality.

► We must choose what to model and what not to model
► There will limitations forced by the formalism
  ► e.g., here we are limited to finite state models
► There will be things we do not understand sufficiently to model
  ► e.g., people

In the words of the The Cure’s Pictures of You:

I’ve been looking so long at these pictures of you
  That I almost believe that they’re real
I’ve been living so long with my pictures of you
  That I almost believe that the pictures are
  All I can feel

Do not do this: the pictures are not real.
Model Checking – Specifications

We are interested in specifying behaviours of systems over time.

- Use Temporal Logic

Specifications are built from:

1. Primitive properties of individual states e.g., “is on”, “is off”, “is active”, “is reading”;
2. Propositional connectives: $\land$, $\lor$, $\neg$;
3. and temporal connectives: e.g., At all times, the system is not simultaneously reading and writing.

If a request signal is asserted at some time, a corresponding grant signal will be asserted within 10 time units.

The exact set of temporal connectives differs across temporal logics.

Logics can differ in how they treat time:

- Linear time vs. Branching time

These differ in reasoning about non-determinism.
Model Checking – Specifications

We are interested in specifying behaviours of systems over time.

▶ Use Temporal Logic

Specifications are built from:

1. Primitive properties of individual states
e.g., “is on”, “is off”, “is active”, “is reading”;

2. propositional connectives $\land, \lor, \neg, \rightarrow$;

3. and temporal connectives: e.g.,
   At all times, the system is not simultaneously reading and writing.
   If a request signal is asserted at some time, a corresponding grant signal will be asserted within 10 time units.
Model Checking – Specifications

We are interested in specifying behaviours of systems over time.

▶ Use Temporal Logic

Specifications are built from:

1. Primitive properties of individual states
e.g., “is on”, “is off”, “is active”, “is reading”;

2. propositional connectives $\land, \lor, \neg, \rightarrow$;

3. and temporal connectives: \textit{e.g.},
At all times, the system is not simultaneously reading and writing.
If a request signal is asserted at some time, a corresponding grant signal will be asserted within 10 time units.

The exact set of temporal connectives differs across temporal logics. Logics can differ in how they treat time:

▶ Linear time vs. Branching time

These differ in reasoning about \textit{non-determinism}. 
Non-determinism

In general, system descriptions are non-deterministic.

A system is non-deterministic when, from some state there are multiple alternative next states the system could transition to.

Non-determinism is good for:

▶ Modelling alternative inputs to the system from its environment (External non-determinism)
▶ Under-specifying the model, allowing it to capture many possible system implementations (Internal non-determinism)
Linear vs. Branching Time

- **Linear Time**
  - Considers paths (sequences of states)
  - If system is non-deterministic, many paths for each initial state
  - Questions of the form:
    - For all paths, does some path property hold?
    - Does there exist a path such that some path property holds?

- **Branching Time**
  - Considers tree of possible future states from each initial state
  - If system is non-deterministic from some state, tree forks
  - Questions can become more complex, *e.g.*, 
    - For all states reachable from an initial state, does there exist an onwards path to a state satisfying some property?
  - Most-basic branching-time logic (CTL) is complementary to most-basic linear-time logic (LTL)
  - Richer branching-time logic (CTL*) incorporates CTL and LTL.
A Taste of LTL – Syntax

LTL = Linear(-time) Temporal Logic

Assume some set $\text{Atom}$ of atomic propositions

Syntax of LTL formulas $\phi$:

$$
\phi, \psi ::= p \mid \neg \phi \mid \phi \lor \psi \mid \phi \land \psi \mid \phi \rightarrow \psi \mid X\phi \mid F\phi \mid G\phi \mid \phi U \psi
$$

where $p \in \text{Atom}$.

Pronunciation:

- $X\phi$ — neXt $\phi$
- $F\phi$ — Future $\phi$
- $G\phi$ — Globally $\phi$
- $\phi U \psi$ — $\phi$ Until $\psi$

Other common connectives: $W$ (weak until), $R$ (release).

Precedence high-to-low: $(X, F, G, \neg), (U), (\land, \lor), \rightarrow$
LTL formulas are evaluated at a position $i$ along a path $\pi$ through the system (a path is a sequence of states connected by transitions).

- An atomic $p$ holds if $p$ is true the state at position $i$.
- The propositional connectives $\neg$, $\land$, $\lor$, $\rightarrow$ have their usual meanings.
- $X\phi$ holds if $\phi$ holds at the next position;
- $F\phi$ holds if there exists a future position where $\phi$ holds;
- $G\phi$ holds if, for all future positions, $\phi$ holds;
- $\phi U \psi$ holds if there is a future position where $\psi$ holds, and $\phi$ holds for all positions prior to that.

This will be made more formal in the next lecture.
A Taste of LTL – Examples

1. \( \mathbf{G} \) \textit{invariant} \\
   \textit{invariant} is true for all future positions
A Taste of LTL – Examples

1. $G \text{ invariant}$
   
   $\text{invariant}$ is true for all future positions

2. $G \neg (read \land write)$
   
   In all future positions, it is not the case that $read$ and $write$
A Taste of LTL – Examples

1. **G invariant**
   
   *invariant* is true for all future positions

2. **G \neg(read \land write)**
   
   In all future positions, it is not the case that *read* and *write*

3. **G(request \rightarrow Fgrant)**
   
   At every position in the future, a *request* implies that there exists a future point where *grant* holds.
A Taste of LTL – Examples

1. $G \text{ invariant}$
   
   invariant is true for all future positions

2. $G \neg (read \land write)$
   
   In all future positions, it is not the case that read and write

3. $G(request \rightarrow Fgrant)$
   
   At every position in the future, a request implies that there
   exists a future point where grant holds.

4. $G(request \rightarrow (request U grant))$
   
   At every position in the future, a request implies that there
   exists a future point where grant holds, and request holds up
   until that point.
A Taste of LTL – Examples

1. \( G \) invariant
   
invariant is true for all future positions

2. \( G \neg(read \land write) \)
   
   In all future positions, it is not the case that read and write

3. \( G(request \rightarrow Fgrant) \)
   
   At every position in the future, a request implies that there exists a future point where grant holds.

4. \( G(request \rightarrow (request \lor grant)) \)
   
   At every position in the future, a request implies that there exists a future point where grant holds, and request holds up until that point.

5. \( G F \) enabled
   
   In all future positions, there is a future position where enabled holds.
A Taste of LTL – Examples

1. \( G \) invariant
   
   invariant is true for all future positions

2. \( G \neg (read \land write) \)
   
   In all future positions, it is not the case that \( read \) and \( write \)

3. \( G(request \rightarrow Fgrant) \)
   
   At every position in the future, a \( request \) implies that there exists a future point where \( grant \) holds.

4. \( G(request \rightarrow (request \lor grant)) \)
   
   At every position in the future, a \( request \) implies that there exists a future point where \( grant \) holds, and \( request \) holds up until that point.

5. \( G F \) enabled
   
   In all future positions, there is a future position where \( enabled \) holds.

6. \( F G \) enabled
   
   There is a future position, from which all future positions have \( enabled \) holding.
Summary

- Introduction to Model Checking (H&R 3.1, 3.2)
  - The Model Checking problem
  - Informal introduction to LTL
- Next time:
  - Formal introduction to LTL.