Solutions to Exercise sheet 1: Propositional Logic

Exercise 1

1. Cats chase mice or birds, but not at the same time.
   This can be represented as: \((M \lor B) \land \neg (M \land B)\)
   where \(M\): Cats chase mice \(B\): Cats chase birds

   \[
   \begin{array}{c|c|c|c|c}
   M & B & M \lor B & \neg (M \land B) & (M \lor B) \land \neg (M \land B) \\
   \hline
   t & t & t & f & f \\
   t & f & t & t & t \\
   f & t & t & t & t \\
   f & f & f & t & f \\
   \end{array}
   \]

2. If it rains the beach will be empty.
   This can be represented as: \(R \rightarrow E\)
   where \(R\): It rains \(E\): Beach is empty

   \[
   \begin{array}{c|c|c}
   R & E & R \rightarrow E \\
   \hline
   t & t & t \\
   t & f & f \\
   f & t & t \\
   f & f & t \\
   \end{array}
   \]

3. If Jane bought a piano today, she either sold her old one or took out a bank loan.
   This can be represented as: \(P \rightarrow S \lor B\)
   where \(P\): Jane bought a piano today \(S\): Jane sold her old piano
   \(B\): Jane took out a bank loan

   \[
   \begin{array}{c|c|c}
   P & S & \lor & B \\
   \hline
   t & t & t \\
   t & f & f \\
   f & t & t \\
   f & f & t \\
   \end{array}
   \]
Exercise 2

The proposition \( P \land (P \rightarrow Q) \) is satisfiable if there is some interpretation which evaluates to \textit{true}. It is valid if all interpretations evaluate to \textit{true}.

\[
P \land (P \rightarrow Q) \\
= P \land (\neg P \lor Q) \\
= (P \land \neg P) \lor (P \land Q) \\
= \bot \lor (P \land Q) \\
= P \land Q
\]

\( P \land Q \) is satisfiable, since it evaluates to \textit{true} when \( P \) is \textit{true} and \( Q \) is \textit{true}. Thus \( P \land (P \rightarrow Q) \) is satisfiable.

\( P \land Q \) is not valid, since it evaluates to \textit{false} when \( P \) is \textit{false} and \( Q \) is \textit{false}. Thus \( P \land (P \rightarrow Q) \) is not valid.

Exercise 3

<table>
<thead>
<tr>
<th>CONNECTIVE</th>
<th>EXPRESSION WITH</th>
<th>ALONE</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \neg )</td>
<td>( p \mid p )</td>
<td></td>
</tr>
<tr>
<td>( \land )</td>
<td>( (p \mid q) \mid (p \mid q) )</td>
<td></td>
</tr>
<tr>
<td>( \lor )</td>
<td>( (p \mid p) \mid (q \mid q) )</td>
<td></td>
</tr>
<tr>
<td>( \rightarrow )</td>
<td>( p \mid (q \mid q) )</td>
<td></td>
</tr>
</tbody>
</table>

Some notes:
- \( p \land q \) is the same as \( \neg (p \mid q) \)
- \( p \lor q \) is the same as \( \neg (\neg p \land \neg q) \)
- \( p \rightarrow q \) is the same as \( \neg p \lor q \)