Exercise sheet 7: Rewriting (Solutions)

Exercise 1

(a) One solution is:

$(\neg p \wedge \neg\neg q) \vee r$	$\neg\neg q$ matches with LHS of $\neg\neg A \Rightarrow A$
$= (\neg p \wedge q) \vee r$	replace with RHS

$exp = (\neg p \wedge \neg\neg q) \vee r$
$sub = \neg\neg q$
$lhs = \neg\neg A$
$rhs = A$
$\phi = \{q/A\}$

(b) One normal form is:

$\neg\ (\ \neg p \wedge (q \vee \neg r))$	
$= \neg\neg p \vee \neg(q \vee \neg r))$	(From rule 2)
$= p \vee \neg(q \vee \neg r))$	(From rule 1)
$= p \vee (\neg q \vee \neg\neg r))$	(From rule 3)
$= p \vee (\neg q \vee r))$	(From rule 1)

Exercise 2

To show that the rule terminates we need some decreasing measure. Could choose:

- Average depth of parse tree decreases
- Number of arithmetic operations decreases
- Number of terms decreases

Exercise 3

Need 2 rewrite rules for critical pairs. We have:

$$p(p(x)) \Rightarrow g(x)$$
$$p(p(x')) \Rightarrow g(x')$$

Recall from lectures that a critical pair is defined as:

$$< rhs_1 \circ \theta,\ (lhs_1\ [rhs_2]) \circ \theta >$$

where $\theta = mgu$ of bit (subpart of lhs_1) and lhs_2.

If we take:

$$\begin{array}{lll} lhs_1 = p(p(x)) & rhs_1 = g(x) & bit = p(x) \\ lhs_2 = p(p(x')) & rhs_2 = g(x') & \theta = \{p(x')/x\} \end{array}$$

the critical pair is:

$$< g(p(x'))\ ,\ p(g(x')) >$$

If we instead take $bit = x$ (so $\theta = \{p(p(x'))/x\}$) then the critical pair is:

$$< g(p(p(x')))\ ,\ g(g(x')) >$$

If we instead take $bit = p(p(x))$ (so $\theta = \{x'/x\}$) then the critical pair is:

$$< g(x')\ ,\ g(x') >$$