Exercise sheet 6:
Unification (Solutions)

Exercise 1

(a) 1. \((X \equiv 2) \land (X \equiv 2)\) (by decompose)
2. \((2 \equiv 2) \land (X \equiv 2)\) (by eliminate)
3. \(X \equiv 2\) (by delete)

Succeeds with \(X=2\)

(b) 1. \((X \equiv 2 + 2) \land (X \equiv 4)\) (by decompose)
2. \((4 \equiv 2 + 2) \land (X \equiv 4)\) (by eliminate)

Fails (by conflict)

(c) 1. \((X \equiv a) \land (Y \equiv g(b)) \land (Y \equiv g(b))\) (by decompose)
2. \((X \equiv a) \land (g(b) \equiv g(b)) \land (Y \equiv g(b))\) (by eliminate)
3. \((X \equiv a) \land (Y \equiv g(b))\) (by delete)

Succeeds with \(X=a\) and \(Y = g(b)\)

(d) 1. \((X \equiv a) \land (b \equiv Y)\) (by decompose)

Fail as target contains a variable.

Exercise 2

(a) 1. \((X \equiv a) \land (b \equiv Y)\) (by decompose)
2. \((X \equiv a) \land (Y \equiv b)\) (by switch)

Succeeds with \(X=a\) and \(Y = b\)

(b) 1. \((X \equiv Y) \land (b \equiv a)\) (by decompose)

Fails

(c) 1. \((X \equiv f(Y)) \land (a \equiv Y)\) (by decompose)
2. \((X \equiv f(Y)) \land (Y \equiv a)\) (by switch)
3. \((X \equiv f(a)) \land (Y \equiv a)\)  \hspace{1em} \text{(by \textit{eliminate})}

Succeeds with \(X \equiv f(a)\) and \(Y \equiv a\).

(d) 1. \((X \equiv f(Y)) \land (g(X) \equiv Y)\)  \hspace{1em} \text{(by \textit{decompose})}
2. \((X \equiv f(Y)) \land (g(f(Y)) \equiv Y)\)  \hspace{1em} \text{(by \textit{coalesce})}
3. \((X \equiv f(Y)) \land (Y \equiv g(f(Y)))\)  \hspace{1em} \text{(by \textit{switch})}

Fails due to occurs check.

(e) 1. \((a + X \equiv a) \land (b \equiv Y)\)  \hspace{1em} \text{(by \textit{decompose})}
2. \((a \equiv a + X) \land (b \equiv Y)\)  \hspace{1em} \text{(by \textit{switch})}

Fails due to occurs check.