Errata for Maths Reasoning Book: 1st edition *

Helen Lowe

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Chapter 1: Introduction

Section 1.1: Why Read This Book?

p.1, lines -10 and -9:

omit commas round "so-called" and "programming language".

p.8, line 11:

omit full stop after syllogism.

Chapter 3: The Internal Structure of Propositions

Section 3.3: The Meaning of Formulae

p.30, line -4:

Omit the words *variable free*. (The occurrence of 'variable free' to describe 'instances' is redundant since instances have already been defined as formulae containing no free variables.)

p.31, lines -12 to -10:

This paragraph (beginning *Recall that* ...) reads less ambiguously if replaced by:
Note also that instances are formed by substituting objects from some set (e.g. 1, 2, 3) for all free variables in a formula, rather than substituting arbitrary terms (e.g. $2+2$, $2+X$).

Section 3.4: Identifying Correct Arguments - Part 2

p.35, line 15:

omit "variable free" in the sentence "However, one of the variable free instances of this is ..."

p.36, line 10:

omit both occurrences of "variable free".

Section 3.5: Summary

p.37, line -4:

Omit comma after "arise".

Chapter 4: Miscellaneous Topics

Section 4.1.1: Variable Functions and Predicate

p.39, lines 5–10 of this section:

Paragraph beginning 'Or suppose...' should be replaced by:

Similarly, we might want to say that equal objects have equal values under the same function. We will want to use a variable function, F, to stand for any unary function.

$$\forall F\ \forall X\ \forall Y\quad X = Y \rightarrow F(X) = F(Y)$$

Section4.1.4: Omega Order Logic

p.41, line -9 to p.42, line 2:

The definition of the expressions using Lambda Calculus - should read as follows (note the addition of free variables in the definition):

1. A constant has no free variables.
2. The free variables of a variable X are $\{X\}$.
3. If s is an expression of type τ containing free variables, $X_1, ..., X_n$, of types, $\tau_1, ..., \tau_n$, respectively, then $\lambda X_1 ... \lambda X_n s$ is an expression of type $\tau_1 \times ... \times \tau_n \mapsto \tau$ whose free variables are those of s minus $\{X_1, ..., X_n\}$
4. If f is an expression of type $\tau_1 \times ... \times \tau_n \mapsto \tau$ and $s_1, ..., s_n$ are expressions of type $\tau_1, ..., \tau_n$, respectively, then $f(s_1, ..., s_n)$ is an expression of type τ whose free variables are the union of those of f and $s_1, ..., s_n$.
5. If p is an expression of type $truth$ containing free variable X then $\forall X\ p$ and $\exists X\ p$ are expressions of type $truth$ whose free variables are those of P minus $\{X\}$.
6. Expressions can only be formed from constants and variables and by application of rules 1–5 above.

Chapter 6

p.66, line -5:

Thoralf (not Thorolf) Skolem.

Chapter 7: Criticisms of Uniform Proof Procedures

Section 7.2: A Resolution Proof and the Combinatorial Explosion

p.85, Figure 7-1

Figure 7-1 should include arc labels as depicted below:

Section 7.3.2: Cheating Techniques

line -1:

Insert Left bracket before "although".

Section 7.5: Alternative Axiomatization

p.92, line 12:

The definition of commutator subgroups should make explicit mention of skolem functions as follows:

$$
\begin{aligned}
&el(commutator\_subgroup(G)) \\
&\qquad = i(el(G)) \circ i(el'(G)) \circ el(G) \circ el'(G)
\end{aligned}
$$

$el(G)$ and $el'(G)$ are rather like skolem functions. We know nothing about their values other than that they are elements of the group G.

Chapter 8: Decision Procedures for Inequalities

Section 8.4.2: An Overview of the Method

p.105, line 1:

This should read:

$$\neg(5 \cdot z \geq 2 \cdot y + 3 \wedge z \leq x - y \wedge \neg 3 \cdot x = 5 \rightarrow 2 \cdot y + 1 < 3)$$

Section 8.4.3: Assigning Types to Skolem Constants

p.105, line 19:

In (a), replace "Eliminate" by "Rewrite".

Chapter 9: Rewrite Rules

Section 9.5.2: Outside In Application

p.122, lines -5 to -1:

The wrong connectives appear in the last few lines.The corrected text is as follows:

> only then can the outermost $\neg$ be brought inside, producing
> $\neg(\neg p \wedge q) \vee \neg r$
>
> The earlier work on $\neg(p \vee \neg q)$ must then be undone with
> $\neg(\neg p \wedge q)$ being rewritten as $\neg\neg p \vee \neg q$ and then $p \vee \neg q$.

Chapter 11: The Productive Use of Failure

Section 11.2: Symbolic Evaluation

p.154, line 19

The first occurrence of x in the second line of the proof (starting centre page) should be a 'nil'. Thus the proof actually is:

$$
\begin{aligned}
&equal(\underline{append(nil, x), x)} \\
&\qquad = equal(\underline{cond(}\underline{nil,} \\
&\qquad\qquad\qquad \underline{cons(car(nil), append(cdr(nil), x)),} \\
&\qquad\qquad\qquad \underline{x),} \\
&\qquad\qquad x) \\
&\qquad = \underline{equal(x, x)} \\
&\qquad = \underline{tt}
\end{aligned}
$$

p.155, line 2:

One of the nested occurrences of 'cons' should be 'cond' (the last but one):

$$
\begin{aligned}
\underline{append(a,b)} = {} & cond(a, \\
& \qquad cons(car(a), \underline{append(cdr(a),b)}), \\
& \qquad b) \\
= {} & cond(a, \\
& \qquad cons(car(a), \\
& \qquad\qquad cond(cdr(a), \\
& \qquad\qquad\qquad cons(car(cdr(a)), \\
& \qquad\qquad\qquad\qquad \underline{append(cdr(cdr(a)),b)}), \\
& \qquad\qquad\qquad b), \\
& \qquad b) \\
= {} & \ldots\ldots\ldots (and\ so\ on)
\end{aligned}
$$

Section 11.3: The Method of Induction

p.156,lines 3–5:

The selection of induction schemata should read as follows:

$$
\begin{aligned}
& P(nil) \wedge \forall X\ \forall X1\, [P(X) \rightarrow P(cons(X1,X))] \rightarrow \forall X\ P(X) \\
& P(nil) \wedge \forall X\ \forall X1\, [P(X1) \wedge P(X) \rightarrow P(cons(X1,X)] \\
& \qquad \rightarrow \forall X\ P(X) \\
& P(nil) \wedge \forall X\ P(cons(X,nil)) \wedge \\
& \qquad \forall X\ \forall X1 \forall X2\, [P(X) \rightarrow P(cons(X1, cons(X2,X)))] \\
& \qquad \rightarrow \forall X\ P(X)
\end{aligned}
$$

Section 11.5: Applications to Arithmetic

p.162, lines 9–10:

Exercise 35 should read as follows:
Exercise 35: Describe how the Boyer-Moore technique would prove the conjecture,
$(X+Y)+Z^2 = X+(Y+Z^2)$

Chapter 12: Formalizing Control Information

Section 12.2.1: Isolation

p.166, lines 13–14

The iff structure should read:

$$F(U_1, ..., U_i, ...U_n) = V \leftrightarrow \bigvee_j U_i = T_i^{-1}(U_1, ..., V, ...U_n)$$

where $\bigvee_j$ indicates that the right hand side of the structure is a disjunction of terms.

p.166 lines -6 and -2, and p.167 line 1 (after diagram:

For '$\log_e x^2 - 1$' read $\log_e(x^2-1)$.

Chapter 16. Herbrand Proof Procedures

Section 16.6: The Soundness and Completeness of Resolution

p.233, line 12 of section:

There is a missing negation symbol (on the $P''n$) in the second clause of the soundness proof: it should read:

2. $C'' \vee \neg P''_1 \vee ... \vee \neg P''_n$

Three lines further down the page the disjointed phrase 'true in I' appears; this is a misprint (omit).

Section 16.6:

p.234, line -14 onwards:

There are some missing scripts (on the $\Re$'s) in the statement of the completeness theorem (theorem 7),and in the preceding paragraph.Corrected version as below:

When these are conjoined to A the result is denoted $\Re(A)$. $\Re(\Re(\Re(...(A)...)))$ n times is denoted $\Re^n(A)$. We will now prove:

Theorem 7:Completeness

If A is an unsatisfiable formula in conjunctive normal form then there is an n such that f is a clause of $\Re^n(A)$

Chapter 17. Pattern Matching

Section 17.5.2:Building-in the Laws of Lambda Calculus

p.251, line 9:

The second of the two equal expressions produced by substitution has a superscript missing. It should read:

$$\{\lambda Z\ Z/F, \sin(X){\cdot}e^X/Y\}$$

Appendix I: Some Artificial Mathematicians Written in PROLOG

p.273, line -2:

omit extraneous "**in our example may be represented as:".

p.281, footnote:

"of" should be "or".

p.282, line 10:

omit comma after "of".

p.283, lines -13 and -10:

the obliques / and // should be \.
Similarly on p.286, lines -3 and -4.

p.284, lines 9–11:

Exercise 69 omits information required for its solution. It should read:
Exercise 69: Represent, using *is_clause*, the clauses from the 'not divides' example of section 10.5 and an extra equality axiom of the form

$$equal(X, Y)\ \&\ not\_div(Y, Z) \rightarrow not\_div(X, Z)$$

Represent, using *interpret*, the models *arith*2 and *arith*3. Run the theorem prover on this example.

The Complete Program

p.285, bottom of page:

There is a missing variable - *Agenda* - in the first sub-goal of the rule 'add-to-agenda' at the bottom of the page. There is also a whole clause missing which should follow the one referred to above. The corrected clause and the omitted one are shown below:

CORRECTED CLAUSE:

```
add_to_agenda([Name | Rest], Agenda, NewAgenda) :-
    \+ in(Name, Agenda), %new
    evaluate(Name, Score),
    insert_into_agenda(Agenda, Score, Name, MidAgenda), %new
    add_to_agenda(Rest, MidAgenda, NewAgenda).
```

OMITTED CLAUSE:

```
add_to_agenda([Name | Rest], Agenda, NewAgenda) :-
    in(Name, Agenda),
    add_to_agenda(Rest, Agenda, Newagenda).
```

The 'Loop Check' clause half way down p.286 also needs replacing with the following clause:

```
/* Loop Check */                    %new
on(Clause, Agenda) :-
    is_clause(Clause, Conse, Ante),
    member(pair(Score, Another), Agenda),
    is_clause(Another, Conse, Ante).
```

Lastly,the very last clause,bottom of p.288,should have a backslash,as opposed to a minus symbol,in its tail:

```
interpret(model1, false, equal(X, Y)) :-
    X \== Y.
```

Appendix IV.Solutions to the Exercise

Solutions to Chapter 4

p.298, Exercise 11:

The base step of the recursive definition of exponentiation should be that X is the successor of zero, viz: $X = s(0)$

Solutions to Chapter 6

p.298, line -9:

In Exercises 16 and 17, the solution tree to exercise 16 is actually given as that for exercise 17 and vice-versa.

Solutions to Chapter 7

p.299, Exercise 20:

axiom 13 should read:

$X \epsilon G$ & $exponent(G, 2) \rightarrow i(X) = X$, and also read $exponent(G, 2)$ in axiom 17.

Solutions to Chapter 9

p301, Exercise 26:

The solutions to exercise 26 erroneously include disjunction symbols as opposed to conjunctions and fail to include brackets: Corrected solutions are:

$\neg\neg A \Rightarrow A$ applies to $(\neg p \wedge \neg\neg q) \vee r$ and gives $(\neg p \wedge q) \vee r$

The values of:

exp is $(\neg p \wedge \neg\neg q) \vee r$,

Solutions to Chapter 10

p.302, Exercise 31:

The solution to Exercise.31 in the text is misleading.The corrected solution is as follows:
Any interpretation of the group axioms and hypotheses of the theorem in which a and b are assigned two unequal values, e.g. boole (see section 3.3.2) where 0 is $+$, $e \mapsto 0$, $a \mapsto 0$, $b \mapsto 1$, $\neg a \mapsto 0$ and not $\neg b \mapsto 1$. $a = b$ is false in this model so $a = b \ \rightarrow$ fails the truth test.

Solutions to Chapter 11

Solution to exercise 35:

For $X + s(Y) + Z$ read $X + Y + Z^2$.

Figure 1: Lush Resolution Search Tree to Depth 2