Solutions to Self-Help Exercise 4: Unification and Rewrite Rules

Exercise 1

(a) 1. $(X \equiv 2) \wedge (X \equiv 2)$ (by *decompose*)
2. $(2 \equiv 2) \wedge (X \equiv 2)$ (by *eliminate*)
3. $X \equiv 2$ (by *delete*)

Succeeds with $X{=}2$

(b) 1. $(X \equiv 2 + 2) \wedge (X \equiv 4)$ (by *decompose*)

Fails

(c) 1. $(X \equiv a) \wedge (Y \equiv g(b)) \wedge (Y \equiv g(b))$ (by *decompose*)
2. $(X \equiv a) \wedge (g(b) \equiv g(b)) \wedge (Y \equiv g(b))$ (by *eliminate*)
3. $(X \equiv a) \wedge (Y \equiv g(b))$ (by *delete*)

Succeeds with $X{=}a$ and $Y = g(b)$

(d) 1. $(X \equiv a) \wedge (b \equiv Y)$ (by *decompose*)

Fail as target contains a variable.

Exercise 2

(a) 1. $(X \equiv a) \wedge (b \equiv Y)$ (by *decompose*)
2. $(X \equiv a) \wedge (Y \equiv b)$ (by *switch*)

Succeeds with $X{=}a$ and $Y = b$

(b) 1. $(X \equiv Y) \land (b \equiv a)$ (by *decompose*)

Fails

(c) 1. $(X \equiv f(Y)) \land (a \equiv Y)$ (by *decompose*)
2. $(X \equiv f(Y)) \land (Y \equiv a)$ (by *switch*)
3. $(X \equiv f(a)) \land (Y \equiv a)$ (by *eliminate*)

Succeeds with $X \equiv f(a)$ and $Y \equiv a$.

(d) 1. $(X \equiv f(Y)) \land (g(X) \equiv Y)$ (by *decompose*)
2. $(X \equiv f(Y)) \land (g(f(Y)) \equiv Y)$ (by *coalesce*)
3. $(X \equiv f(Y)) \land (Y \equiv g(f(Y)))$ (by *switch*)

Fails due to occurs check.

(e) 1. $(a + X \equiv a) \land (b \equiv Y)$ (by *decompose*)
2. $(a \equiv a + X) \land (b \equiv Y)$ (by *switch*)

Fails due to occurs check.

Exercise 3

(a) One solution is:

$(\neg p \land \neg\neg q) \lor r$ $\quad$ $\neg\neg q$ matches with LHS of $\neg\neg A \Rightarrow A$
$= (\neg p \land q) \lor r$ $\quad$ replace with RHS

$exp = (\neg p \land \neg\neg q) \lor r$
$sub = \neg\neg q$
$lhs = \neg\neg A$
$rhs = A$
$\phi = \{q/A\}$

(b) One normal form is:

$\neg\ (\ \neg p \land (q \lor \neg r))$
$= \neg\neg p \lor \neg(q \lor \neg r))$ (From rule 2)
$= p \lor \neg(q \lor \neg r))$ (From rule 1)
$= p \lor (\neg q \lor \neg\neg r))$ (From rule 3)

$= p \vee (\neg q \vee r))$ (From rule 1)

Exercise 4

To show that the rule terminates we need some decreasing measure. Could choose:

- Average depth of parse tree decreases
- Number of arithmetic operations decreases
- Number of terms decreases

Exercise 5

Need 2 rewrite rules for critical pairs. We have:

$$\begin{aligned} p(p(x)) &\Rightarrow g(x) \\ p(p(x')) &\Rightarrow g(x') \end{aligned}$$

Recall from lectures that a critical pair is defined as:

$$< rhs_1 \circ \theta, (lhs_1\ [rhs_2]) \circ \theta >$$

where $\theta = mgu$ of bit (subpart of lhs_1) and lhs_2.

If we take:

$$\begin{array}{lll} lhs_1 = p(p(x)) & rhs_1 = g(x) & bit = p(x) \\ lhs_2 = p(p(x')) & rhs_2 = g(x') & \theta = \{p(x')/x\} \end{array}$$

the critical pair is:

$$< g(p(x'))\ ,\ p(g(x')) >$$

If we instead take $bit = x$ (so $\theta = \{p(p(x'))/x\}$) then the critical pair is:

$$< g(p(p(x')))\ ,\ g(g(x')) >$$

If we instead take $bit = p(p(x))$ (so $\theta = \{x'/x\}$) then the critical pair is:

$$< g(x')\ ,\ g(x') >$$