Self-Help Exercise 4:
Unification and Rewrite Rules

Exercise 1

Apply one way unification (matching) to the following pairs of expressions. Outline the behaviour of the matching process. Determine whether the process fails or succeeds in each case. In the case of success determine the resulting substitution.

	pattern	**target**
(a)	$X = X$	$2 = 2$
(b)	$X = X$	$2 + 2 = 4$
(c)	$p(f(X, Y), Y)$	$p(f(a, g(b)), g(b))$
(d)	$X = b$	$a = Y$

Exercise 2

Apply two way unification to the following pairs of expressions. Outline the behaviour of the unification process. Determine whether the process fails or succeeds in each case. In the case of success determine the resulting substitions.

	exp1	**exp2**
(a)	$X = b$	$a = Y$
(b)	$X = b$	$Y = a$
(c)	$p(X, a)$	$p(f(Y), Y)$
(d)	$p(X, g(X))$	$p(f(Y), Y)$
(e)	$(a + X) + b$	$a + Y$

Exercise 3

Consider the following rewrite rules:

$$(1)\ \neg\neg A \Rightarrow A$$
$$(2)\ \neg(A \wedge B) \Rightarrow \neg A \vee \neg B$$
$$(3)\ \neg(A \vee B) \Rightarrow \neg A \wedge \neg B$$

(a) Apply one of these to the following formula. Say what *exp, sub, lhs, rhs* and ϕ are.

$$(\neg p \wedge \neg\neg q) \vee r$$

(b) Find one normal form of the following formula by applying rewrite rules to it until no more apply, *i.e* show one complete branch of the search space.

$$\neg(\neg p \wedge (q \vee \neg r))$$

Exercise 4

Show that the application of the rule:

$$X \star Y + X \star Z \Rightarrow X \star (Y + Z)$$

will terminate.

Exercise 5

Find all critical pairs of the following rewrite rule with itself:

$$parent(parent(X)) \Rightarrow grandparent(X)$$

What new rewrite rule(s) do these critical pair(s) suggest?