

## Solutions to Self-Help Exercise 1: Propositional Logic

### Exercise 1

1. Cats chase mice or birds, but not at the same time.

This can be represented as:  $(M \vee B) \wedge \neg(M \wedge B)$

where  $M$ : Cats chase mice       $B$ : Cats chase birds

$M$	$B$	$M \vee B$	$\neg(M \wedge B)$	$(M \vee B) \wedge \neg(M \wedge B)$
$t$	$t$	$t$	$f$	$f$
$t$	$f$	$t$	$t$	$t$
$f$	$t$	$t$	$t$	$t$
$f$	$f$	$f$	$t$	$f$

2. If it rains the beach will be empty.

This can be represented as:  $R \longrightarrow E$

where  $R$ : It rains       $E$ : Beach is empty

$R$	$E$	$R \longrightarrow E$
$t$	$t$	$t$
$t$	$f$	$f$
$f$	$t$	$t$
$f$	$f$	$t$

3. If Jane bought a piano today, she either sold her old one or took out a bank loan.

This can be represented as:  $P \longrightarrow S \vee B$

where  $P$ : Jane bought a piano today       $S$ : Jane sold her old piano

$B$ : Jane took out a bank loan

$P$	$S$	$L$	$P \longrightarrow S \vee B$
$t$	$t$	$t$	$t$
$t$	$t$	$f$	$t$
$t$	$f$	$t$	$t$
$t$	$f$	$f$	$f$
$f$	$t$	$t$	$t$
$f$	$t$	$f$	$t$
$f$	$f$	$t$	$t$
$f$	$f$	$f$	$t$

## Exercise 2

The proposition  $P \wedge (P \longrightarrow Q)$  is satisfiable if there is some interpretation which evaluates to *true*. It is valid if all interpretations evaluate to *true*

$$\begin{aligned}
& P \wedge (P \longrightarrow Q) \\
= & P \wedge (\neg P \vee Q) \\
= & (P \wedge \neg P) \vee (P \wedge Q) \\
= & f \vee (P \wedge Q) \\
= & P \wedge Q
\end{aligned}$$

$P \wedge Q$  is satisfiable, since it evaluates to *true* when  $P$  is *true* and  $Q$  is *true*. Thus  $P \wedge (P \longrightarrow Q)$  is satisfiable.

$P \wedge Q$  is not valid, since it evaluates to *false* when  $P$  is *false* and  $Q$  is *false*. Thus  $P \wedge (P \longrightarrow Q)$  is not valid.

## Exercise 3

CONNECTIVE	EXPRESSION WITH   ALONE
$\neg$	$p \mid \text{p}$
$\wedge$	$(p \mid q) \mid (p \mid q)$
$\vee$	$(p \mid p) \mid (q \mid q)$
$\longrightarrow$	$p \mid (q \mid q)$

Some notes:

- $p \wedge q$  is the same as  $\neg (p \mid q)$
- $p \vee q$  is the same as  $\neg(\neg p \wedge \neg q)$
- $p \longrightarrow q$  is the same as  $\neg p \vee q$