Advances in Programming Languages
APL7: Haskell, Types and Classes

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Semester 2 Week 4
Some Types in Haskell

This is the first of four lectures about some features of types and typing in Haskell types, specifically:

- Type classes
- Constructor classes, multiparameter type classes
- Monads and interaction with the outside world
- Encapsulating stateful computation
Outline

1  Types

2  Types in Object-Oriented Languages

3  Haskell Curry

4  Type Classes in Haskell

5  Examples of Haskell Type Classes

6  Closing
Some types

A selection of types from some languages.

C/C++

int, long, float, unsigned int, char
int [], char*, char&, int(*)(float, char)

OCaml

int, int64, bool, char, string, unit
string*, string, int list, bool array
int->int, int->string->char, 'a list -> 'a list

Java

Object, byte[], boolean
StringBuffer, LinkedList, TreeSet, ArrayList<String>
IllegalPathStateException, BeanContextServiceRevokedListener
What do people do with types?

- Type checking
- Static type checking
- Dynamic type checking
- Type annotation
- Type inference
- Structural typing
- Nominative typing
- Subtyping
- Duck typing
- Effect types
- Soft typing
- Gradual typing
- Dynamic types
- Blame typing
What is a type system?

A *type system* is a well-defined subset $T$ of programs such that:

$$P \in T \implies \text{Execute}(P) \models \phi$$

(read: “if $P$ is in $T$ then Execute($P$) satisfies $\phi$”)

where $\text{Execute}(P)$ is the behaviour of $P$ when (compiled and) run, and $\phi$ is some desired property of that behaviour.

For example,

$$T = \text{“well-typed Java programs”}$$

$$\phi = \text{“methods are always correctly invoked”}$$

Slogan: *Well-typed programs cannot go wrong.*  
[Robin Milner, 1978]
Read on to find out more…

... and lots more

Outline

1. Types
2. Types in Object-Oriented Languages
3. Haskell Curry
4. Type Classes in Haskell
5. Examples of Haskell Type Classes
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Java is serious about abstraction

Java works almost entirely through class-based object-oriented programming; it encourages the use of abstract classes through inheritance and interfaces; and it does not expose the private workings of classes and packages.

Java is serious about typing

Java has strong static typing: all programs are checked for type-correctness at compile-time. Bytecode is checked again when classes are loaded, by the bytecode verifier, before execution. The introduction of generics extended the power of the type system.

Even so, things do not always go as well as one might hope...
Recall subtyping

Subtyping is a well-established part of the object-oriented paradigm: an object in a subclass can stand in for an object in a superclass.

Sometimes known as Liskov’s *principle of substitutivity*:

> properties that can be proved using the specification of an object’s presumed type should hold even though the object is actually a subtype of that type

[Liskov and Wing, 1994]

We have already seen this in the context of program specification and verification.
Subtyping arrays in Java

Java has subtyping: a value of one type may be used at any more general type. So String $\leq$ Object, and every String is an Object.

```java
String[] a = { "Hello" }; // A small string array
Object[] b = a; // Now a and b are the same array
b[0] = Boolean.FALSE; // Drop in a Boolean object
String s = a[0]; // Oh, dear
System.out.println(s.toUpperCase()); // This isn't going to be pretty
```

This compiles without error or warning: in Java, if $S \leq T$ then $S[] \leq T[]$. Except that it isn’t. So every array assignment gets a runtime check.
Subtype variance

The issue here is with *parameterized types* like `String[]` and `List<Object>;` or in Haskell *Maybe* `a` and `(a,b)→(b,a).

Suppose some type `A⟨X⟩` depends on type `X`, and types `S ≤ T`. Then the dependency is:

- **Covariant** if `A⟨S⟩ ≤ A⟨T⟩`  
  e.g. pair `A⟨X⟩= X * X`
- **Contravariant** if `A⟨S⟩ ≥ A⟨T⟩`  
  e.g. test `A⟨X⟩= X→bool`
- **Invariant** if neither of these holds.  
  e.g. array `A⟨X⟩= X[]`

For example, in the *Scala* language, type parameters can be annotated with variance information: `List[+T]`, `Function[−S,+T]`

In Java, arrays are typed as if they were covariant. But they aren’t.

see also *parameter covariance* in Eiffel
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One problem is that subtyping is crucial to OO programming, but unfortunately:
- subtyping is not inheritance;
- it’s also extremely hard to get right.
Fixing object subtyping has been a busy research topic for several years. You can see this by observing that the type declared for the max method in the Java collections class has gone from:

```
public static Object max(Collection coll)
```

which always returns an `Object`, whatever is stored in the collection, to:

```
public static <T extends Object & Comparable<? super T>> T max(Collection<? extends T> coll)
```

and it might still throw a `ClassCastException`. (Java 7, 2010?)

This is not a criticism: the new typing is more flexible, it saves on explicit downcasts, and the Java folks do know what they are doing.
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Haskell Brooks Curry, 1900–1982
Logician
Curry-Howard correspondence

Propositions as Types

<table>
<thead>
<tr>
<th>A and B</th>
<th>A × B</th>
<th>∀x.A(x)</th>
<th>Πx.A(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A or B</td>
<td>A + B</td>
<td>∃y.B(y)</td>
<td>Σy.B(y)</td>
</tr>
<tr>
<td>A ⇒ B</td>
<td>A → B</td>
<td>∀X.X ⇒ X</td>
<td>ΛX.X → X</td>
</tr>
<tr>
<td>True</td>
<td>1</td>
<td>Proofs</td>
<td>Programs</td>
</tr>
<tr>
<td>False</td>
<td>0</td>
<td>Proof rewriting</td>
<td>Program execution</td>
</tr>
</tbody>
</table>

The Coq proof assistant is built on the correspondence between proofs and terms, leading to features like *computational reflection* and *program extraction*. Also, the first machine-verified proof of the four-colour theorem.
Currying

\[ A \times B \to C \equiv A \to B \to C \]
\[ (A \& B) \Rightarrow C \equiv A \Rightarrow (B \Rightarrow C) \]

Left to right is *currying*  
Right to left is *uncurrying*.

If we had some ham, we could have ham and eggs, if we had any eggs.
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What have classes ever done for us?

Object-oriented languages employ classes, inheritance, and class hierarchy for a range of reasons:

- Substitutability
- Modularity
- Encapsulation
- Abstraction
- Polymorphism
- Code reuse
- ...

Haskell’s *type classes* are quite different, but do provide some similar benefits.
Object-oriented code is *polymorphic* when it can be used with objects from different classes:

```java
Shape[] shapeArray;
...
for (Shape s : shapeArray) // For every shape in the array ...
{ s.draw(); } // ... invoke its "draw" method.
```

Each `Shape s` may actually be a `Square`, `Circle` or other implementation of `Shape`, each with its own implementation of `draw`.

These implementations may be entirely different, and possibly incompatible: consider `Picture.draw()` and `Cowboy.draw()`. 
Ad-hoc vs. Parametric polymorphism

These implementations may be entirely different, and possibly incompatible: consider `Picture.draw()` and `Cowboy.draw()`.

Christopher Strachey named this *ad-hoc polymorphism*. By contrast, *parametric polymorphism* allows code to have the same action across many types of data.

Parametric polymorphism arrived in Java 5 and C# 2.0 as *generics*, now extensively used in the standard libraries of both languages.

Note that C++ *templates* can achieve a similar effect (and many others), but at the cost of duplicating code during compilation. The ideal for parametric polymorphism is that because the action is the same, the executing code should be the same too.
class `Eq a where`
    
    `==` :: `a -> a -> Bool`

instance `Eq Int where`
    
    `i == j = eqInt i j`

instance `(Eq a) => Eq [a] where`
    
    `[] == [] = True; (x:xs) == (y:ys) = (x == y) && (xs == ys)`

member :: `Eq a => a -> [a] -> Bool`
member `x [] = False`
member `x (y:ys) = (x == y) || member x ys`

P. Wadler and S. Blott.
How to make ad-hoc polymorphism less ad-hoc.
Proc. POPL ’89, pp. 60–76.
Pass the dictionary

Type classes can be implemented by *dictionary-passing*. You write:

\[
\text{below} :: \textbf{Num} \ n \Rightarrow \ n \to n \to n \\
\text{below} \ x \ y = y - x
\]

The compiler can turn that into:

\[
\text{below} :: \textbf{Num} \ n \to n \to n \to n \\
\text{below} \ d \ x \ y = (\_\_\_) \ d \ y \ x
\]

Here \((d :: \textbf{Num} \ n)\) is an additional parameter, a *dictionary* of all the operations that make type \(n\) an instance of class \textbf{Num}.

This need not be an expensive translation: subsequent optimisations may well then inline and even eliminate the dictionary if all the types can be determined in advance.
Use the types

When type classes appear, the action selected depends on the types of all parties involved; not just arguments, or the first argument. This opens up some flexibility in just how that action is chosen.

\[
\text{below} :: \text{Num} \ n \Rightarrow n \rightarrow n \rightarrow n \\
\text{below} \ x \ y = y - x
\]

\[
\text{read} :: \text{Read} \ a \Rightarrow \text{String} \rightarrow a \\
\text{read} \ "12" + 3.4
\]

In some cases, the compiler may have to work unexpectedly hard to work out what types, and hence dictionary choices, to make.
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Some type classes immediately present themselves as opportunities for overloading:

- Equality: `Eq`
- Order: `Ord`
- Print and scan: `Show` and `Read`
- Iteration: `Enum`, `Bounded`, `Ix`

Not least, the numeric classes:

- `Num`, `Fractional`, `Real`, `Integral`, `Fractional`, `Floating`, `RealFrac`, `RealFloat`

Remember, those are just the classes. The types matching them are `Float`, `Double`, `Int`, `Integer`, `Rational = Ratio Integer`, `Complex Double`, ...
Somewhat unexpectedly, the ingenious applications of type classes go far, far beyond this.

- Pretty-printing
- Modular arithmetic

```haskell
class Modular s a | s -> a where modulus :: s -> a
```

- Phantom types: `data T a = String`
- Arithmetic in the type system: `class Add a b ab`
- SK combinators, logic programming, Turing completeness...
It's fun to have fun, but you have to know how
QuickCheck

prop_Insert x xs = ordered xs ==> ordered (insert x xs)

Main> quickCheck prop_Insert
OK: passed 100 tests

QuickCheck has no privileged access to the compiler: it uses type classes to obtain the right random generators, for the right number of arguments, for every test.

K. Claessen and John Hughes
QuickCheck: A lightweight tool for random testing of Haskell programs
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For next time, read the following paper and set of slides.

- **P. Wadler and S. Blott.**
  How to make ad-hoc polymorphism less ad-hoc.
  *Proc. POPL ’89*, pp. 60–76.

- **S. L. Peyton Jones.**
  Wearing the hair shirt: a retrospective on Haskell
  Invited talk at POPL 2003.
Further Reading

If you are interested in type classes, and in particular how they can be efficiently implemented, read these.

L. Augustsson
Implementing Haskell Overloading
*Proc. FPCA ’93*

J. Peterson and M. Jones
Implementing Type Classes
*Proc. PLDI ’93*

C. V. Hall, K. Hammond, S. L. Peyton Jones, and P. L. Wadler
Type classes in Haskell,
*Proc. ESOP ’94*