

Advances in Programming Languages

APL7: Haskell, Types and Classes

Ian Stark

School of Informatics
The University of Edinburgh

Monday 1 February
Semester 2 Week 4



Some Types in Haskell

This is the first of four lectures about some features of types and typing in Haskell types, specifically:

- Type classes
- Constructor classes, multiparameter type classes
- Monads and interaction with the outside world
- Encapsulating stateful computation

Outline

- 1 Types
- 2 Types in Object-Oriented Languages
- 3 Haskell Curry
- 4 Type Classes in Haskell
- 5 Examples of Haskell Type Classes
- 6 Closing

Some types

A selection of types from some languages.

C/C++

int, long, float, unsigned int, char
int [], char*, char&, int(*) (float, char)

OCaml

int, int64, bool, char, string, unit
string*string, int list, bool array
int->int, int->string->char, 'a list -> 'a list

Java

Object, **byte[]**, **boolean**
StringBuffer, LinkedList, TreeSet, ArrayList<String>
IllegalPathStateException, BeanContextServiceRevokedListener

What do people do with types?

- Type checking
- Static type checking
- Dynamic type checking
- Type annotation
- Type inference
- Structural typing
- Nominative typing
- Subtyping
- Duck typing
- Effect types
- Soft typing
- Gradual typing
- Dynamic types
- Blame typing

What is a type system?

A *type system* is a well-defined subset T of programs such that:

$$P \in T \implies \text{Execute}(P) \models \phi$$

(read: “if P is in T then $\text{Execute}(P)$ satisfies ϕ ”)

where $\text{Execute}(P)$ is the behaviour of P when (compiled and) run, and ϕ is some desired property of that behaviour.

For example,

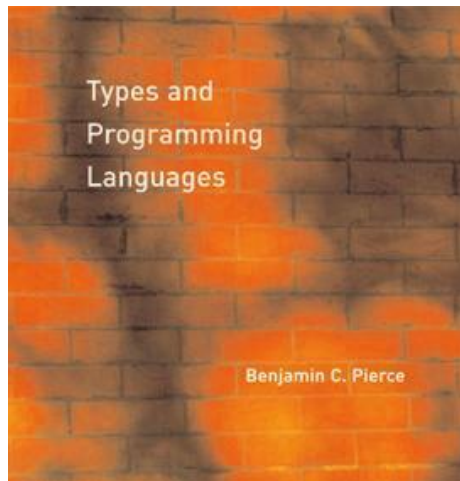
T = “well-typed Java programs”

ϕ = “methods are always correctly invoked”

Slogan: *Well-typed programs cannot go wrong.*

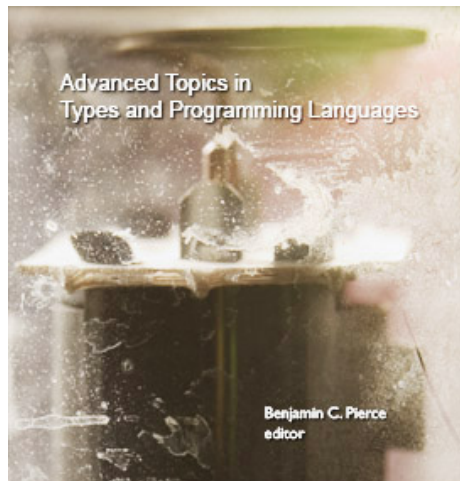
[Robin Milner, 1978]

Read on to find out more...



Benjamin C. Pierce.
*Types and Programming
Languages.*
MIT Press, 2002.

... and lots more



- 📄 Benjamin C. Pierce, editor. *Advanced Topics in Types and Programming Languages*. MIT Press, 2005.

Outline

- 1 Types
- 2 Types in Object-Oriented Languages**
- 3 Haskell Curry
- 4 Type Classes in Haskell
- 5 Examples of Haskell Type Classes
- 6 Closing

Java is serious about abstraction

Java works almost entirely through class-based object-oriented programming; it encourages the use of abstract classes through inheritance and interfaces; and it does not expose the private workings of classes and packages.

Java is serious about typing

Java has strong static typing: all programs are checked for type-correctness at compile-time. Bytecode is checked again when classes are loaded, by the *bytecode verifier*, before execution. The introduction of *generics* extended the power of the type system.

Even so, things do not always go as well as one might hope...

Recall subtyping

Subtyping is a well-established part of the object-oriented paradigm: an object in a subclass can stand in for an object in a superclass.

Sometimes known as Liskov's *principle of substitutivity*:

properties that can be proved using the specification of an object's presumed type should hold even though the object is actually a subtype of that type

[Liskov and Wing, 1994]

We have already seen this in the context of program specification and verification.

Subtyping arrays in Java

Java has subtyping: a value of one type may be used at any more general type. So `String` \leq `Object`, and every `String` is an `Object`.

Not all is well with Java types

```
String[] a = { "Hello" };           // A small string array
Object[] b = a;                     // Now a and b are the same array
b[0] = Boolean.FALSE;               // Drop in a Boolean object
String s = a[0];                    // Oh, dear
System.out.println(s.toUpperCase()); // This isn't going to be pretty
```

This compiles without error or warning: in Java, if $S \leq T$ then $S[] \leq T[]$.
Except that it isn't. So every array assignment gets a runtime check.

Subtype variance

The issue here is with *parameterized types* like `String[]` and `List<Object>`; or in Haskell `Maybe a` and `(a,b) -> (b,a)`.

Suppose some type $A\langle X \rangle$ depends on type X , and types $S \leq T$. Then the dependency is:

Covariant if $A\langle S \rangle \leq A\langle T \rangle$

e.g. pair $A\langle X \rangle = X * X$

Contravariant if $A\langle S \rangle \geq A\langle T \rangle$

e.g. test $A\langle X \rangle = X \rightarrow \text{bool}$

Invariant if neither of these holds.

e.g. array $A\langle X \rangle = X[]$

For example, in the `Scala` language, type parameters can be annotated with variance information: `List[+T]`, `Function[-S, +T]`.

In Java, arrays are typed as if they were covariant. But they aren't.

see also parameter covariance in Eiffel

Typing in OO languages

Ideally, an statically-checked object-oriented language should have a type system that is

Typing in OO languages

Ideally, an statically-checked object-oriented language should have a type system that is

- (a) usable, and

Typing in OO languages

Ideally, an statically-checked object-oriented language should have a type system that is

- (a) usable, and
- (b) correct.

Typing in OO languages

Ideally, an statically-checked object-oriented language should have a type system that is

- (a) usable, and
- (b) correct.

Building such type systems is a continuing challenge.

Typing in OO languages

Ideally, an statically-checked object-oriented language should have a type system that is

- (a) usable, and
- (b) correct.

Building such type systems is a continuing challenge.

One problem is that subtyping is crucial to OO programming, but unfortunately:

Typing in OO languages

Ideally, an statically-checked object-oriented language should have a type system that is

- (a) usable, and
- (b) correct.

Building such type systems is a continuing challenge.

One problem is that subtyping is crucial to OO programming, but unfortunately:

- subtyping is not inheritance;

(really, it's not)

Really, it's not

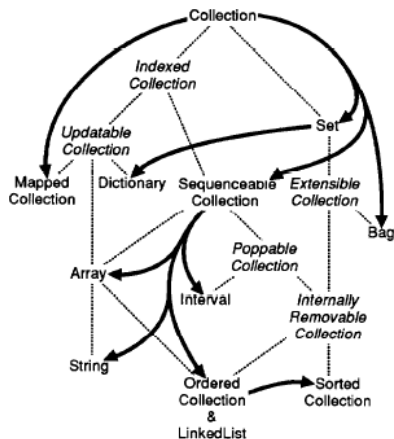


Figure 5: Interfaces versus Inheritance



W. R. Cook.

Interfaces and specifications for the Smalltalk-80 collection classes.

Proc. OOPSLA '92, pp. 1–15.



W. R. Cook, W. Hill, and P. S. Canning.

Inheritance is not subtyping.

Proc. POPL '90, pp. 125–135.

Typing in OO languages

Ideally, an statically-checked object-oriented language should have a type system that is

- (a) usable, and
- (b) correct.

Building such type systems is a continuing challenge.

One problem is that subtyping is crucial to OO programming, but unfortunately:

- subtyping is not inheritance;

Typing in OO languages

Ideally, an statically-checked object-oriented language should have a type system that is

- (a) usable, and
- (b) correct.

Building such type systems is a continuing challenge.

One problem is that subtyping is crucial to OO programming, but unfortunately:

- subtyping is not inheritance;
- it's also extremely hard to get right.

How hard?

Fixing object subtyping has been a busy research topic for several years.

You can see this by observing that the type declared for the `max` method in the Java collections class has gone from: (Java 1.2, 1998)

How hard?

Fixing object subtyping has been a busy research topic for several years.

You can see this by observing that the type declared for the `max` method in the Java `collections` class has gone from: (Java 1.2, 1998)

```
public static Object max(Collection coll)
```

which always returns an `Object`, whatever is stored in the collection, to:

How hard?

Fixing object subtyping has been a busy research topic for several years.

You can see this by observing that the type declared for the `max` method in the Java collections class has gone from: (Java 1.2, 1998)

```
public static Object max(Collection coll)
```

which always returns an `Object`, whatever is stored in the collection, to:

```
public static <T extends Object & Comparable<? super T>> T  
max(Collection<? extends T> coll)
```

How hard?

Fixing object subtyping has been a busy research topic for several years.

You can see this by observing that the type declared for the `max` method in the Java collections class has gone from: (Java 1.2, 1998)

```
public static Object max(Collection coll)
```

which always returns an `Object`, whatever is stored in the collection, to:

```
public static <T extends Object & Comparable<? super T>> T  
max(Collection<? extends T> coll)
```

and it might *still* throw a `ClassCastException`. (Java 7, 2010?)

How hard?

Fixing object subtyping has been a busy research topic for several years.

You can see this by observing that the type declared for the `max` method in the Java collections class has gone from: (Java 1.2, 1998)

```
public static Object max(Collection coll)
```

which always returns an `Object`, whatever is stored in the collection, to:

```
public static <T extends Object & Comparable<? super T>> T  
max(Collection<? extends T> coll)
```

and it might *still* throw a `ClassCastException`. (Java 7, 2010?)

This is not a criticism: the new typing is more flexible, it saves on explicit downcasts, and the Java folks do know what they are doing.

Outline

- 1 Types
- 2 Types in Object-Oriented Languages
- 3 Haskell Curry**
- 4 Type Classes in Haskell
- 5 Examples of Haskell Type Classes
- 6 Closing

Haskell Brooks Curry



Haskell Brooks Curry, 1900–1982
Logician

Curry-Howard correspondence

Propositions as Types

A and B	$A \times B$	$\forall x.A(x)$	$\Pi x.A(x)$
A or B	$A + B$	$\exists y.B(y)$	$\Sigma y.B(y)$
$A \Rightarrow B$	$A \rightarrow B$	$\forall X.X \Rightarrow X$	$\Lambda X.X \rightarrow X$
True	1	Proofs	Programs
False	0	Proof rewriting	Program execution

The *Coq* proof assistant is built on the correspondence between proofs and terms, leading to features like *computational reflection* and *program extraction*.
Also, the first machine-verified proof of the four-colour theorem.

Currying

$$\begin{array}{ccc} A \times B \rightarrow C & \cong & A \rightarrow B \rightarrow C \\ (A \ \& \ B) \Rightarrow C & \Leftrightarrow & A \Rightarrow (B \Rightarrow C) \end{array}$$

Left to right is *currying*

Right to left is *uncurrying*.

If we had some ham, we could have ham and eggs, if we had any eggs.

Outline

- 1 Types
- 2 Types in Object-Oriented Languages
- 3 Haskell Curry
- 4 Type Classes in Haskell**
- 5 Examples of Haskell Type Classes
- 6 Closing

What have classes ever done for us?

Object-oriented languages employ classes, inheritance, and class hierarchy for a range of reasons:

- Substitutability
- Modularity
- Encapsulation
- Abstraction
- Polymorphism
- Code reuse
- ...

Haskell's *type classes* are quite different, but do provide some similar benefits.

Ad-hoc vs. Parametric polymorphism

Object-oriented code is *polymorphic* when it can be used with objects from different classes:

```
Shape[] shapeArray;  
...  
for (Shape s : shapeArray) // For every shape in the array ...  
{ s.draw(); }              // ... invoke its "draw" method.
```

Each `Shape s` may actually be a `Square`, `Circle` or other implementation of `Shape`, each with its own implementation of `draw`.

These implementations may be entirely different, and possibly incompatible: consider `Picture.draw()` and `Cowboy.draw()`.

Ad-hoc vs. Parametric polymorphism

...

These implementations may be entirely different, and possibly incompatible: consider `Picture.draw()` and `Cowboy.draw()`.

Christopher Strachey named this *ad-hoc polymorphism*. By contrast, *parametric polymorphism* allows code to have the same action across many types of data.

Parametric polymorphism arrived in Java 5 and C# 2.0 as *generics*, now extensively used in the standard libraries of both languages.

Note that C++ *templates* can achieve a similar effect (and many others), but at the cost of duplicating code during compilation. The ideal for parametric polymorphism is that because the action is the same, the executing code should be the same too.

Not such ad-hoc polymorphism

class Eq a where

(==) :: a -> a -> Bool

instance Eq Int where

i == j = eqInt i j

instance (Eq a) => Eq [a] where

[] == [] = True; (x:xs) == (y:ys) = (x == y) && (xs == ys)

member :: Eq a => a -> [a] -> Bool

member x [] = False

member x (y:ys) = (x == y) || member x ys



P. Wadler and S. Blott.

How to make ad-hoc polymorphism less ad-hoc.

Proc. POPL '89, pp. 60–76.

Pass the dictionary

Type classes can be implemented by *dictionary-passing*. You write:

```
below :: Num n => n -> n -> n
below x y = y - x
```

The compiler can turn that into:

```
below :: Num n -> n -> n -> n
below d x y = (-) d y x
```

Here $(d :: \text{Num } n)$ is an additional parameter, a *dictionary* of all the operations that make type n an instance of class **Num**.

This need not be an expensive translation: subsequent optimisations may well then inline and even eliminate the dictionary if all the types can be determined in advance.

Use the types

When type classes appear, the action selected depends on the types of all parties involved; not just arguments, or the first argument.

This opens up some flexibility in just how that action is chosen.

```
below :: Num n => n -> n -> n
```

```
below x y = y - x
```

```
read :: Read a => String -> a
```

```
read "12" + 3.4
```

In some cases, the compiler may have to work unexpectedly hard to work out what types, and hence dictionary choices, to make.

Outline

- 1 Types
- 2 Types in Object-Oriented Languages
- 3 Haskell Curry
- 4 Type Classes in Haskell
- 5 Examples of Haskell Type Classes**
- 6 Closing

Natural overloaders

Some type classes immediately present themselves as opportunities for overloading:

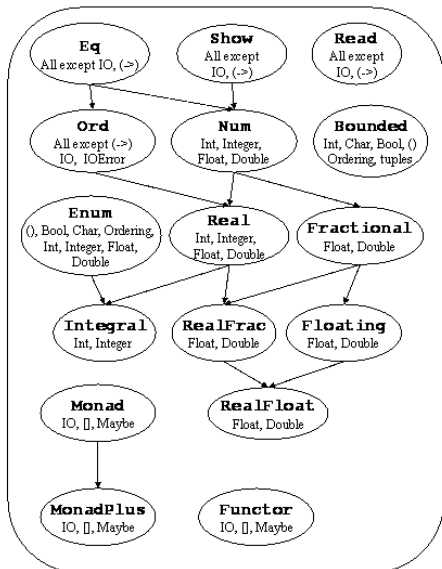
- Equality: **Eq**
- Order: **Ord**
- Print and scan: **Show** and **Read**
- Iteration: **Enum**, **Bounded**, **Ix**

Not least, the numeric classes:

- **Num**, **Fractional**, **Real**, **Integral**, **Fractional**, **Floating**, **RealFrac**, **RealFloat**

Remember, those are just the classes. The types matching them are **Float**, **Double**, **Int**, **Integer**, **Rational = Ratio Integer**, **Complex Double**, ...

Standard Haskell Classes



But wait! There's more

Somewhat unexpectedly, the ingenious applications of type classes go far, far beyond this.

- Pretty-printing
- Modular arithmetic

[Kiselyov & Shan 2004]

```
class Modular s a | s -> a where modulus :: s -> a
```

- Phantom types: **data** T a = **String**
- Arithmetic in the type system: **class** Add a b ab
- SK combinators, logic programming, Turing completeness...

It's fun to have fun, but you have to know how



But seriously

QuickCheck

```
prop_Insert x xs = ordered xs ==> ordered (insert x xs)
```

```
Main> quickCheck prop_Insert
```

```
OK: passed 100 tests
```

QuickCheck has no privileged access to the compiler: it uses type classes to obtain the right random generators, for the right number of arguments, for every test.



K. Claessen and John Hughes

QuickCheck: A lightweight tool for random testing of Haskell programs

Proc. ICFP 2000, pp. 268–279.

Outline

- 1 Types
- 2 Types in Object-Oriented Languages
- 3 Haskell Curry
- 4 Type Classes in Haskell
- 5 Examples of Haskell Type Classes
- 6 Closing**

For next time, read the following paper and set of slides.



P. Wadler and S. Blott.

How to make ad-hoc polymorphism less ad-hoc.

Proc. POPL '89, pp. 60–76.



S. L. Peyton Jones.

Wearing the hair shirt: a retrospective on Haskell

Invited talk at POPL 2003.

Further Reading

If you are interested in type classes, and in particular how they can be efficiently implemented, read these.



L. Augustsson

Implementing Haskell Overloading

Proc. FPCA '93



J. Peterson and M. Jones

Implementing Type Classes

Proc. PLDI '93



C. V. Hall, K. Hammond, S. L. Peyton Jones, and P. L. Wadler

Type classes in Haskell,

Proc. ESOP '94