

# Tests, Backtracking, and Recursion

Artificial Intelligence Programming in Prolog

Lecture 3

30/09/04

# Re-cap

- A Prolog program consists of **predicate definitions**.
- A predicate denotes a property or relationship between objects.
- Definitions consist of **clauses**.
- A clause has a **head** and a **body** (**Rule**) or **just a head** (**Fact**).
- A head consists of a **predicate name** and **arguments**.
- A clause body consists of a conjunction of **terms**.
- Terms can be **constants**, **variables**, or **compound terms**.
- We can set our program **goals** by typing a command that unifies with a clause head.
- A goal unifies with clause heads in order (top down).
- **Unification** leads to the **instantiation** of variables to values.
- If any variables in the initial goal become instantiated this is reported back to the user.

# Correction: Re-trying Goals

- When a question is asked with a variable as an argument (e.g. `greet(Anybody) .`) we can ask the Prolog interpreter for multiple answers using: `;`

```
greet(hamish):- write('How are you doin, pal?').  
greet(amelia):- write('Awfully nice to see you!').
```

```
| ?- greet(Anybody) .  
      How are you doin, pal?  
      Anybody = hamish? ;  
      Awfully nice to see you!  
      Anybody = amelia? ;  
      no
```

- `;` fails the last clause used and searches down the program for another that matches.
- It then performs all the tasks contained within the body of the new clause and returns the new value of the variable.

# Tests

- When we ask Prolog a question we are asking for the interpreter to prove that the statement is true.

`?- 5 < 7, integer(bob) .`

`yes` = the statement can be proven.

`no` = the proof failed because either

- the statement does not hold, or
- the program is broken.

`Error` = there is a problem with the question or program.

`*nothing*` = the program is in an infinite loop.

- We can ask about:
  - Properties of the database: `mother(jane, alan) .`
  - Built-in properties of individual objects: `integer(bob) .`
  - Absolute relationships between objects:
    - Unification: `=/2`
    - Arithmetic relationships: `<, >, =<, >=, ==, +, -, *, /`

# Arithmetic Operators

- Operators for arithmetic and value comparisons are built-in to Prolog
    - = always accessible / don't need to be written
  - Comparisons: <, >, =<, >=, **:=** (equals), **=\=** (not equals)
    - = Infix operators: go between two terms.
    - =</2 is used
      - 5 =< 7. (infix)
      - =<(5, 7). (prefix) ← all infix operators can also be prefixed
  - Equality is different from unification
    - =/2 checks if two terms unify
    - :=/2 compares the arithmetic value of two expressions
- |         |                     |                        |
|---------|---------------------|------------------------|
| ?- X=Y. | ?- X:=Y.            | ?-X=4,Y=3, X+2 := Y+3. |
| yes     | Instantiation error | X=4, Y=3? yes          |

# Arithmetic Operators (2)

- Arithmetic Operators:  $+$ ,  $-$ ,  $*$ ,  $/$ 
  - = Infix operators but can also be used as prefix.
  - Need to use `is/2` to access result of the arithmetic expression otherwise it is treated as a term:

```
|?- X = 5+4.
```

```
X = 5+4 ?
```

```
yes
```

(Can X unify with 5+4?)

```
|?- X is 5+4.
```

```
X = 9 ?
```

```
yes
```

(What is the result of 5+4?)

- Mathematical precedence is preserved:  $/$ ,  $*$ , before  $+$ ,  $-$
- Can make compound sums using round brackets
  - Impose new precedence
  - Inner-most brackets first

```
| ?- X is (5+4)*2.
```

```
X = 18 ?
```

```
yes
```

# Tests within clauses

- These operators can be used within the body of a clause:

- To manipulate values,

```
sum(X, Y, Sum) :-
```

```
    Sum is X+Y.
```

- To distinguish between clauses of a predicate definition

```
bigger(N, M) :-
```

```
    N < M, write('The bigger number is '), write(M).
```

```
bigger(N, M) :-
```

```
    N > M, write('The bigger number is '), write(N).
```

```
bigger(N, M) :-
```

```
    N == M, write('Numbers are the same').
```

# Backtracking

```
|?- bigger(5,4).
```

```
    ↑   ↑   ↑  
bigger(N,M):-
```

```
    N < M,
```

```
    write('The bigger number is '), write(M).
```

```
bigger(N,M):-
```

```
    N > M,
```

```
    write('The bigger number is '), write(N).
```

```
bigger(N,M):-
```

```
    N == M,
```

```
    write('Numbers are the same').
```



# Backtracking

```
|?- bigger(5,4) .  
      ↑      ↑      ↑  
bigger(5,4):-  
    5 < 4, ← fails  
      write('The bigger number is '), write(M) .  
bigger(N,M):-  
    N > M,  
      write('The bigger number is '), write(N) .  
bigger(N,M):-  
    N == M,  
      write('Numbers are the same').
```

Backtrack

# Backtracking

```
|?- bigger(5,4) .  
      ↑   ↑   ↑  
bigger(N,M):-  
    N < M,  
    write('The bigger number is '), write(M) .  
      ↓   ↓   ↓  
bigger(5,4):-  
    5 > 4,  
    write('The bigger number is '), write(N) .  
bigger(N,M):-  
    N == M,  
    write('Numbers are the same') .
```

# Backtracking

|?- bigger(5,4) .

bigger(N,M) :-

N < M,

write('The bigger number is '), write(M) .

bigger(5,4) :-

5 > 4, ← succeeds, go on with body.

write('The bigger number is '), write(5) .

The bigger number is 5

yes

|?-

Reaches full-stop  
= clause succeeds

# Backtracking

```

|?- bigger(5,5).  ← If our query only matches the final clause

bigger(N,M):-
    N < M,
    write('The bigger number is '), write(M).
bigger(N,M):-
    N > M,
    write('The bigger number is '), write(N).
bigger(5,5):-
    5 == 5,  ← Is already known as the first two clauses failed.
    write('Numbers are the same').
  
```

The diagram illustrates the backtracking process. Three vertical arrows point upwards from the query `bigger(5,5)` to the first two clauses of the `bigger` predicate. The first arrow points to the first clause `bigger(N,M):- N < M, ...`, the second to the second clause `bigger(N,M):- N > M, ...`, and the third to the third clause `bigger(5,5):- 5 == 5, ...`. This shows that the first two clauses failed, and the third clause was eventually matched.

## Backtracking

`|?- bigger(5,5) .`    ← If our query only matches the final clause

```

bigger(N,M):-
    N < M,
    write('The bigger number is '), write(M) .

```

```

bigger(N,M):-
    N > M,
    write('The bigger number is '), write(N) .

```

**`bigger(5,5):-`**

← Satisfies the same conditions.

**`write('Numbers are the same') .`**

Numbers are the same

yes

Clauses should be ordered according to specificity  
 Most specific at top ↔ Universally applicable at bottom

# Reporting Answers

`|?- bigger(5,4).`

← Question is asked

The bigger number is 5

← Answer is written to terminal

yes

← Succeeds but answer is lost

- This is fine for checking what the code is doing but not for using the proof.



`|?- bigger(6,4), bigger(Answer,5).`

Instantiation error!

- To report back answers we need to
  - put an uninstantiated variable in the query,
  - instantiate the answer to that variable when the query succeeds,
  - pass the variable all the way back to the query.

# Passing Back Answers

- To report back answers we need to
  1. put an **uninstantiated variable** in the query,

| ?- bigger(6,4,**Answer**), bigger(**Answer**,5,New\_answer) .


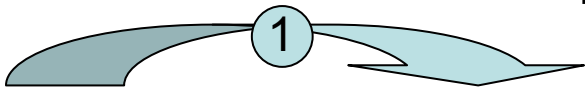
bigger(**X**,Y,Answer) :- **X**>Y, Answer = X.  
bigger(X,**Y**,Answer) :- X=<**Y**, Answer = Y.

2. instantiate the answer to that variable when the query succeeds,
3. pass the variable all the way back to the query.

# Head Unification

- To report back answers we need to
  1. put an **uninstantiated variable** in the query,

| ?- bigger(6,4,**Answer**),bigger(**Answer**,5,**New\_answer**) .



```
bigger(X,Y,X) :- X>Y.  
bigger(X,Y,Y) :- X=<Y.
```

Or, do steps 2 and 3 in one step by naming the variable in the head of the clause the same as the correct answer.

**= head unification**



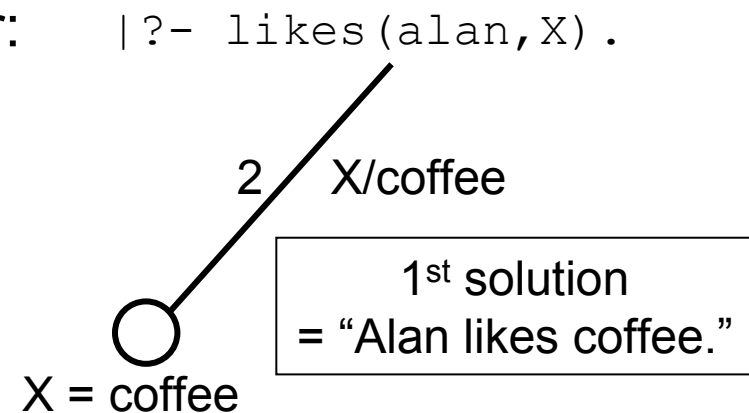
# Satisfying Subgoals

- Most rules contain calls to other predicates in their body. These are known as **Subgoals**.
- These subgoals can match:
  - facts,
  - other rules, or
  - the same rule = **a recursive call**

```
1) drinks(alan,beer) .  
2) likes(alan,coffee) .  
3) likes(heather,coffee) .  
  
4) likes(Person,Drink) :-  
    drinks(Person,Drink) .    ← a different subgoal  
5) likes(Person,Somebody) :-  
    likes(Person,Drink) ,      ← recursive subgoals  
    likes(Somebody,Drink) .    ←
```

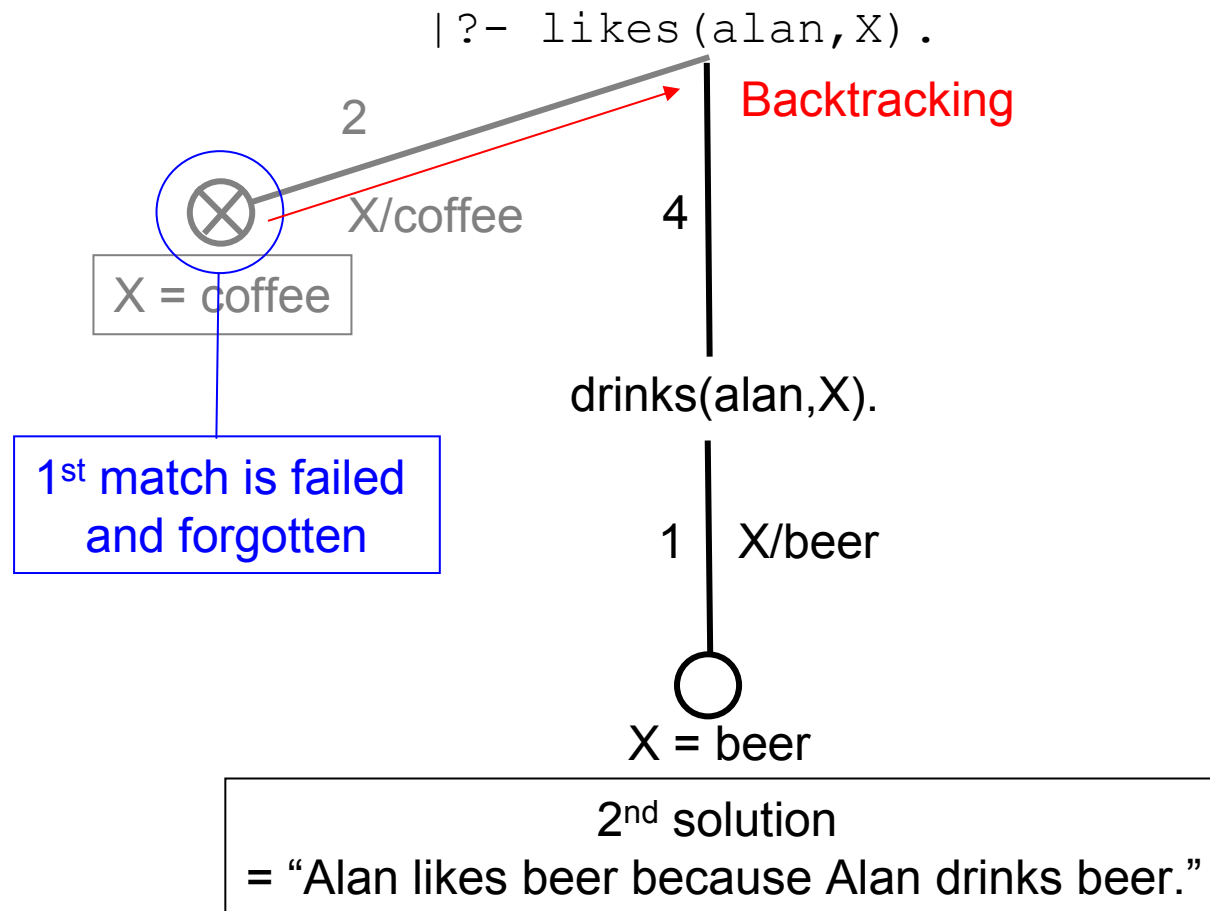
# Representing Proof using Trees

- To help us understand Prolog's proof strategy we can represent its behaviour using **AND/OR trees**.
  1. Query is the top-most point (node) of the tree.
  2. Tree grows downwards (looks more like roots!).
  3. Each branch denotes a subgoal.
    1. The branch is labelled with the number of the matching clause and
    2. any variables instantiated when matching the clause head.
  4. Each branch ends with either:
    1. A successful match ○ ,
    2. A failed match ⊗ , or
    3. Another subgoal.



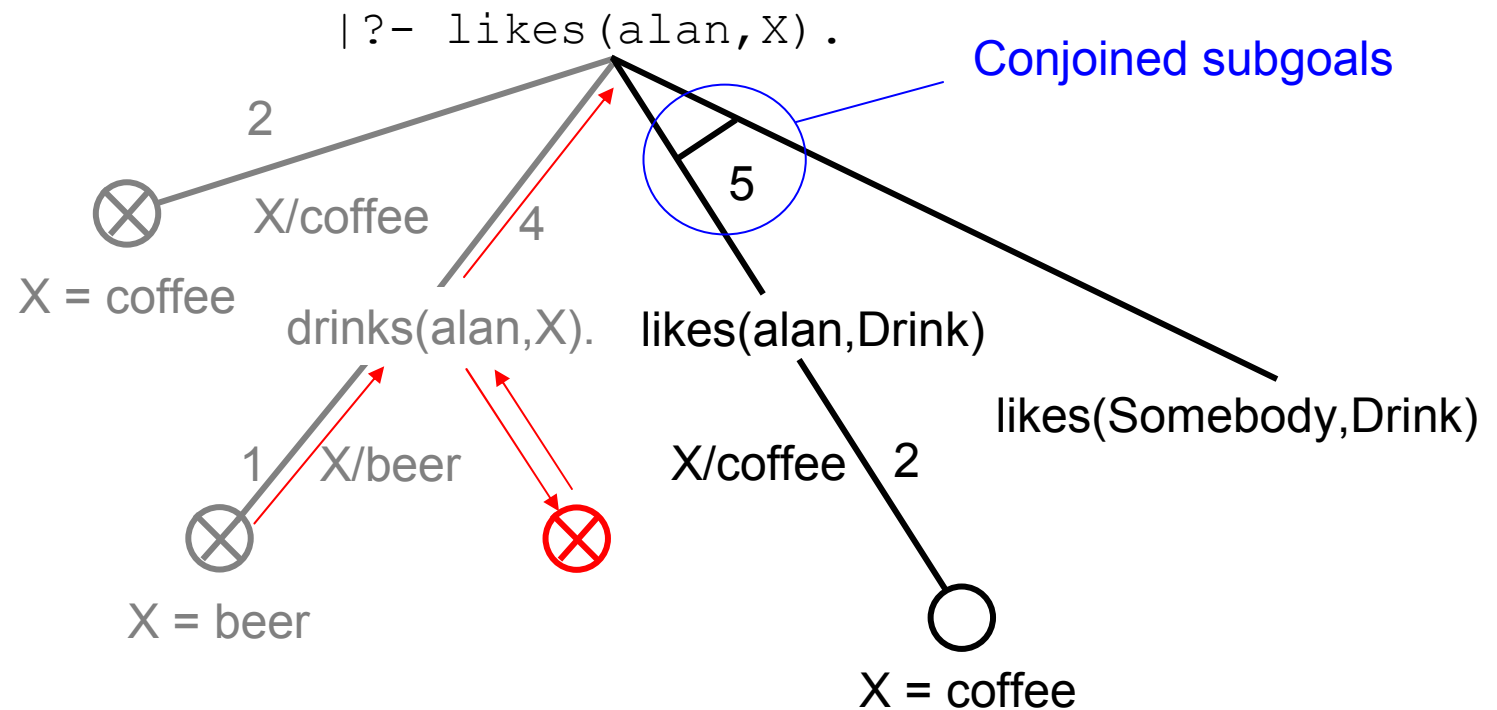
## Representing Proof using Trees (2)

- Using the tree we can see what happens when we ask for another match ( ; )



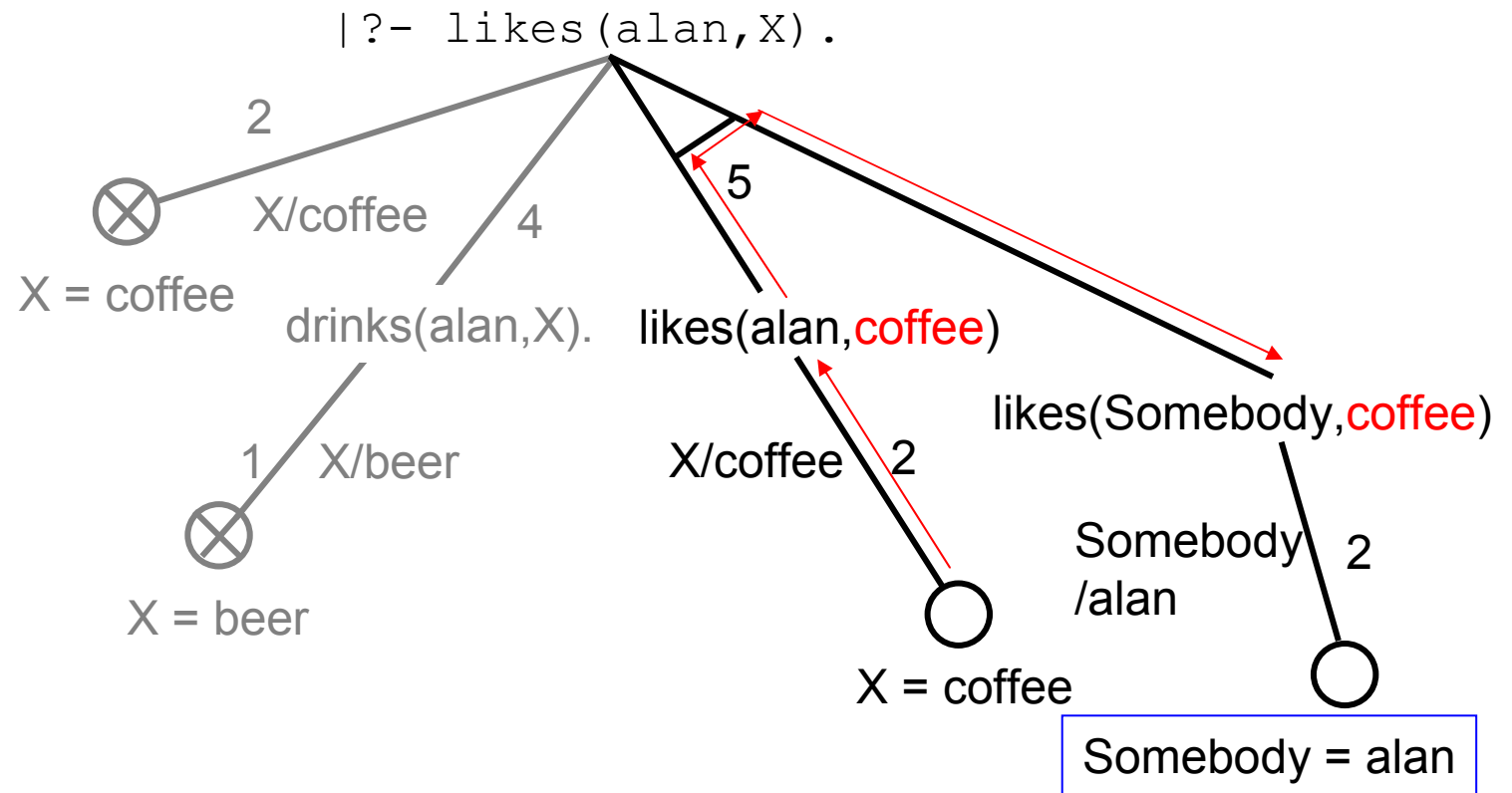
# Recursion using Trees

- When a predicate calls itself within its body we say the clause is **recursing**



# Recursion using Trees (2)

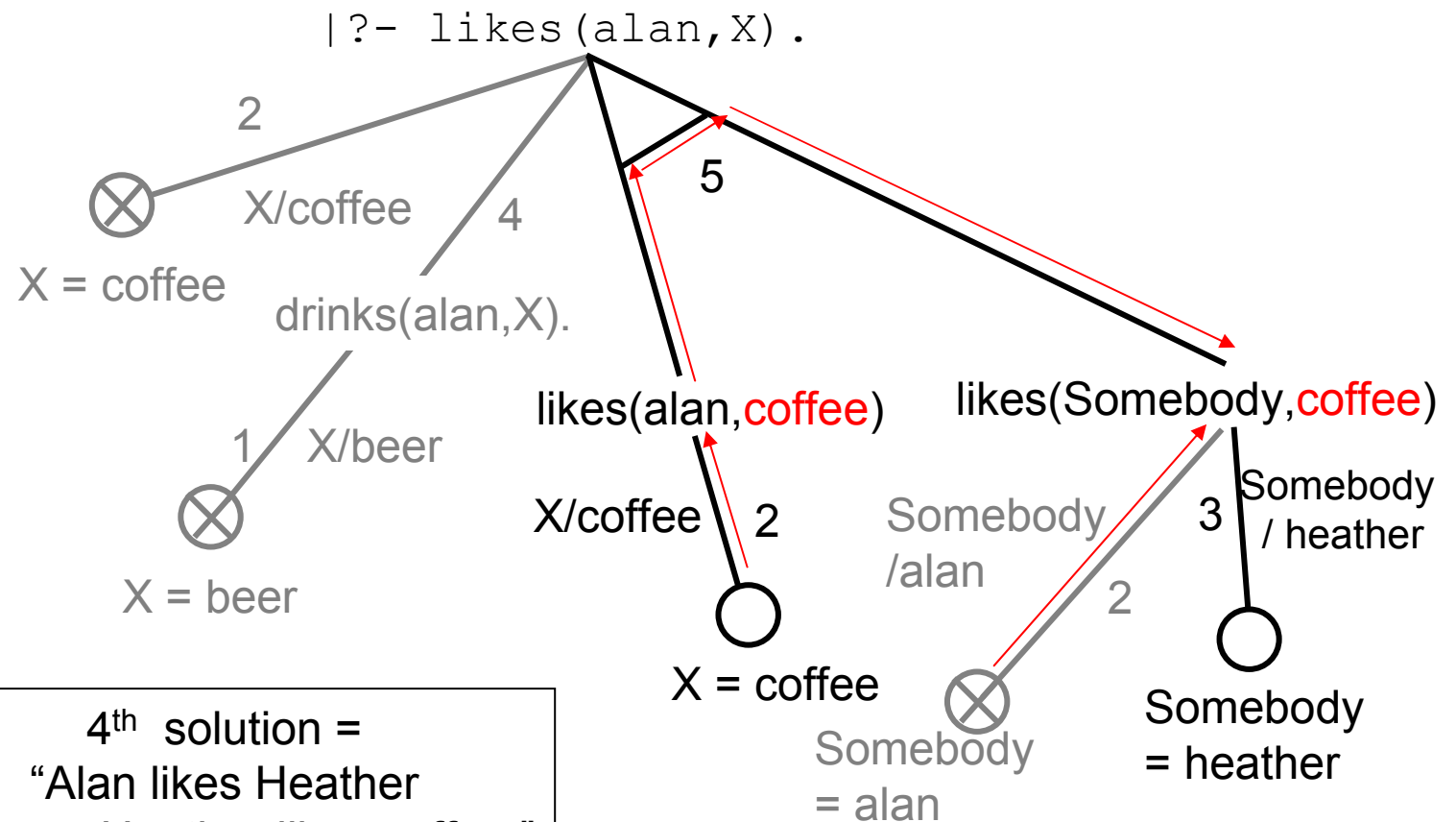
- When a predicate calls itself within its body we say the clause is **recursing**



3<sup>rd</sup> solution = "Alan likes Alan because Alan likes coffee."

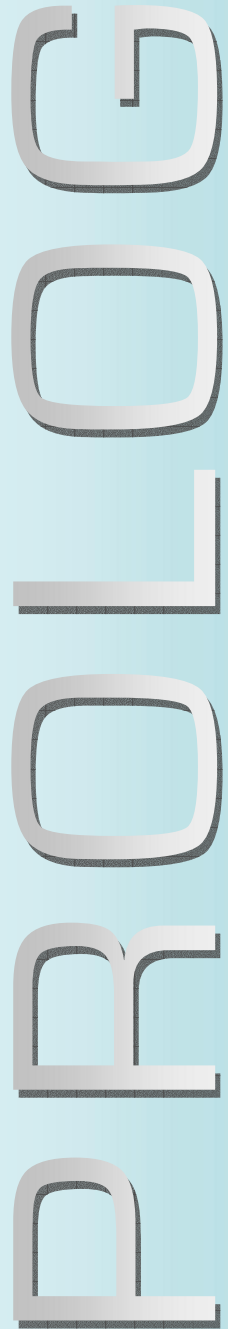
# Recursion using Trees (3)

- When a predicate calls itself within its body we say the clause is **recursing**



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- [illegible]



# The central ideas of Prolog

- SUCCESS/FAILURE
  - any computation can “**succeed**” or “**fail**”, and this is used as a ‘**test**’ mechanism.
- MATCHING
  - any two data items can be compared for similarity, and values can be bound to variables in order to allow a match to succeed.
- SEARCHING
  - the whole activity of the Prolog system is to search through various options to find a combination that succeeds.
    - Main search tools are [backtracking](#) and [recursion](#)
- BACKTRACKING
  - when the system fails during its search, it returns to previous choices to see if making a different choice would allow success.