Situation Calculus

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(slides courtesy of Robert Wilensky)

Using Logic to Plan

- Need
  - A way of representing the world.
  - A way of representing the goal.
  - A way of representing how actions change the world.
- We haven't said much about the last.
  - Difficulty is, after an action, new things are true, and some previously true facts are no longer true.

Situations

- Idea:
  - Introduce a notion of *situations*.
  - State facts about situations.
    - By relativizing predications to situations.
    - E.g., instead of saying just **On(A,B)**, say (somehow) '**On(A,B)** in situation S_0'.
  - Actions
    - will be performed in a situation, and
    - will produce new situations with new facts.

Representing Predications Relative to a Situation

- Can add a place for a situation to each predicate that can change.
  - E.g., instead of **On(A,B)**, write **On(A,B,S_0)**
- Alternatively, introduce a predicate **Holds**; make **On**, etc., *functions*:
  - E.g., **Holds(On(A,B),S_0)**
  - What do things like **On(A,B)** now mean?
    - Either a category of situations, in which **A** is on **B**, or a set of those situations.

How This Will Work

- Before some action, we might have in our KB:
  On(A,B,S_0)
  On(B,Table,S_0)
  ....
- After an action that moves A to the table, say, we add
  Clear(B,S_1)
  On(A,Table,S_1)
- All these propositions are true, but we have dealt with the issue of change, by keeping track of what is true when.

Same Thing, Slightly Different Notation

- Before :
  Holds(On(A,B),S_0)
  Holds(On(B,Table),S_0)
  ...
- After, add
  Holds(Clear(B),S_1)
  Holds(On(A,Table),S_1)

Representing Actions

- Need to represent:
  - Results of doing an action
  - Conditions that need to be in place to perform an action.
- For convenience, we will define *functions* to abbreviate actions:
  - E.g., **Move(A,B)** denotes the *action type* of moving **A** onto **B**.
  - These are action *types*, because actions themselves are specific to time, etc.
- Now, introduce a *function* **Result**, designating `the situation resulting from doing an action type in some situation'.
  - E.g., **Result(Move(A,B),S_0)** means `the situation resulting from doing an action of type **Move(A,B)** in situation **S_0**'.

How This Works

- Keep in mind that things like
  Result(Move(A,B),S_0)
  are *terms*, and denote *situations*.
  - They can appear anywhere we would expect a situation.
- So we can say things like
  S_1=Result(Move(A,B),S_0)
  On(A,B,Result(Move(A,B),S_0))
  On(A,B,S_1)
  (Alternatively, **Holds(On(A,B),Result(Move(A,B),S_0))**, etc.)

Axiomatizing Actions

- Now we can describe the results of actions, together with their preconditions.
- E.g., 'If nothing is on x and y, then one can move x to on top of y, in which case x will then be on y.'

  ∀x,y,s Clear(x,s) ∧ Clear(y,s)

  ⇒ On(x,y,Result(Move(x,y),s))
- Alternatively:

  ∀x,y,s

  Holds(Clear(x),s) ∧ Holds(Clear(y),s)

  ⇒ Holds(On(x,y), Result(Move(x,y),s))
- This is an *effect axiom*.
  - Although it includes a precondition as well.

Situation Calculus

- This approach is called the *situation calculus*.
- We axiomitize all our actions, then use a general theorem prover to prove that a situation exists in which our goal is true.
- The actions in the proof would comprise our plan.

A Very Simple Example

- KB:

  On(A,Table,S_0)

  On(B,C,S_0)

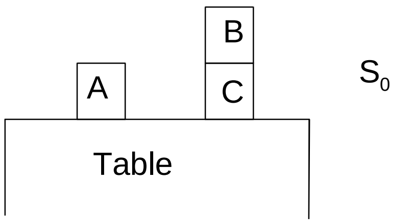
  On(C,Table,S_0)

  Clear(A,S_0)

  Clear(B,S_0)

  and axioms about actions, etc.
- Goal:

  ∃s' On(A,B,s')



What happens?

- We try to prove **On(A,B,s')**
  - Find axiom

    ∀x,y,s Clear(x,s) ∧ Clear(y,s)

    ⇒ On(x,y,Result(Move(x,y),s))
  - By chaining, e.g., goal would be true if we could prove **Clear(A,s) ∧ Clear(B,s)**.
  - But both are true in **S_0**, so we can conclude **On(A,B,Result(Move(A,B),S_0))**
- We are done!
- We look in the proof and see only one action, **Move(A,B)**, which is executed in situation **S_0**, so this is our plan.

Tougher Example: Same Initial World, Harder Goal

- KB:

  On(A,Table,S_0)

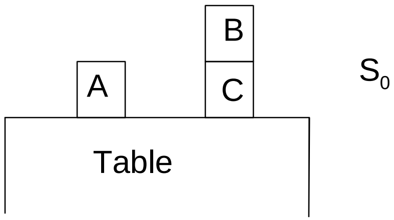
  On(B,C,S_0)

  On(C,Table,S_0)

  Clear(A,S_0)

  Clear(B,S_0)

  and axioms about actions, etc.



Goal:

∃s' On(A,B,s') ∧ On(B,C,s')

(Intuitively, really not harder: B already on C, and we just showed how to make A on B.)

With Goal ∃s'On(A,B,s') ∧ On(B,C,s')

- Suppose we try to prove the first subgoal, **On(A,B,s')**.
  - Use same axiom

    ∀x,y,s Clear(x,s) ∧ Clear(y,s)

    ⇒ On(x,y, Result(Move(x,y),s))
  - Again, by chaining, we can conclude **On(A,B,Result(Move(A,B),S_0))**.
  - Abbreviating **Result(Move(A,B),S_0)** as **S_1**, we have **On(A,B,S_1)**.
- Substituting in our other subgoal makes that **On(B,C,S_1)**. If this is true, we're done.
- But we have no reason to believe this is true!
- Sure, **On(B,C,S_0)**, but how does the planner know this is still true, i.e., **On(B,C,S_1)**?
- In fact, it doesn't, so it fails to find an answer!

The Frame Problem

- We have failed to express the fact that everything that isn't changed by an action stays the same.
- Can fix by adding *frame axioms*. E.g.:

  ∀x,y,z,s

  Clear(x,s) ⇒ Clear(x, Result(Paint(x,y),s))

  ...
- There are *lots* of these!
- Is this a big problem?

Better Frame Axioms

- Can fix with neater formulation:

  ∀x,y,z,s,a

  On(x,y,s) ∧ ¬(a=Move(x,z) ∧ ¬y=z)

  ⇒ On(x,y,Result(a,s))
- Can combine with effect axioms to get *successor-state axioms*:

  ∀x,y,z,s,a

  On(x,y,Result(a,s)) ⇔

  On(x,y,s) ∧ ¬(a=Move(x,z) ∧ ¬y=z)

  ∨ Clear(x,s) ∧ Clear(y,s) ∧ a=Move(x,y)

Note How This Helps Our Example

- Want to prove
  $On(B,C,Result(Move(A,B),S_0))$
  given that **$On(B,C,S_0)$**
  Axiom says
  $On(x,y,Result(a,s)) \Leftrightarrow$
  $On(x,y,s) \wedge \neg(a=Move(x,z) \wedge \neg y=z)$
  $\vee\ Clear(x,s) \wedge Clear(y,s) \wedge a=Move(x,y)$
- So need to show
  $On(B,C,S_0) \wedge \neg(Move(A,B)=Move(B,z) \wedge \neg C=z)$

The first conjunct is in the KB; the second is true because the actions are clearly different.

Frame problem partially solved

- This solves the representational part of the frame problem.
- Still have to compute that everything that was true that wasn't changed is still true. Inefficient (as is general theorem proving).
- Solution: Special purpose representations, special purpose algorithms, called *Planners*.