Formalizing Communicative Actions

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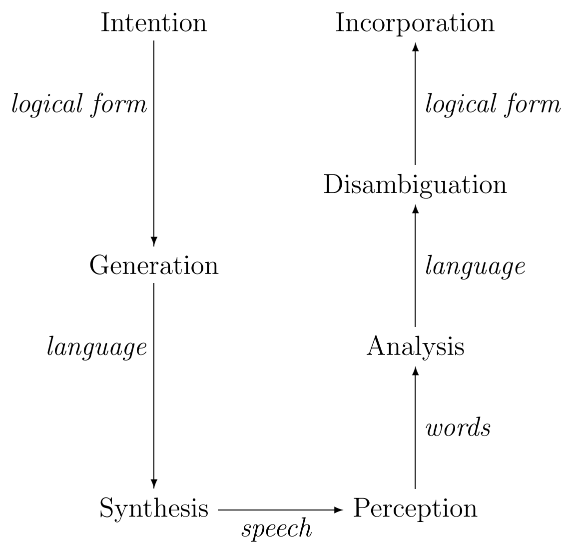
A Complex Model of Communication



A Simple Model of Communication

Assume communication works by direct transmission of logical formulae.

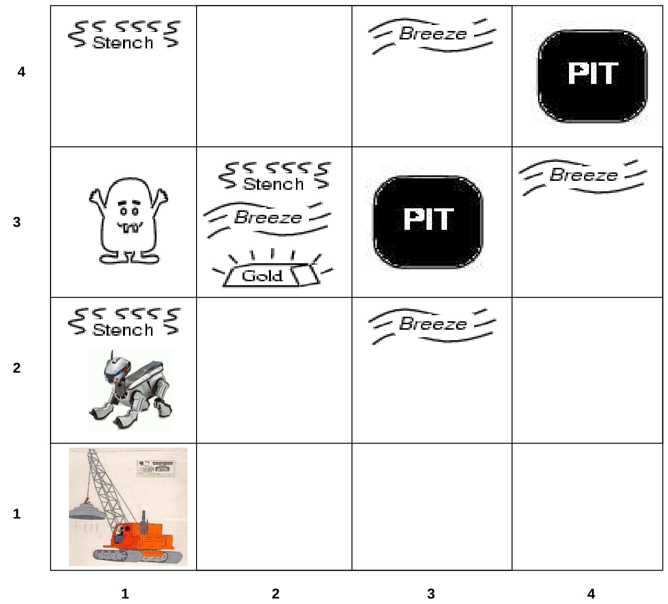
Side-step issue of translation to/from speech, *etc.*

Inform: Speaker can convey information to hearer.

Query: Speaker can ask hearer for information.

Request: Speaker can ask hearer to perform action.

Multi-Agent Wumpus World



Abilities of Agents

- Crane agent can move gold, but cannot sense.
- Dog agent can sense, but cannot move gold.
- They need to cooperate to win game.
- Cooperation requires communication.

Some Regular Agent Actions

$$\begin{aligned}\texttt{action:}\ & Forward(ag, sq_1, sq_2)\\ \texttt{precondition:}\ & At(ag, sq_1) \wedge Heading(ag, dir) \wedge\\ & Next(sq_1, dir, sq_2) \wedge OK(sq_2)\\ \texttt{add:}\ & At(ag, sq_2)\\ \texttt{delete:}\ & At(ag, sq_1)\end{aligned}$$

$$\begin{aligned}\texttt{action:}\ & Right(ag, dir_1, dir_2)\\ \texttt{precondition:}\ & Ninety(dir_1, dir_2) \wedge Heading(ag, dir_1)\\ \texttt{add:}\ & Heading(ag, dir_2)\\ \texttt{delete:}\ & Heading(ag, dir_1)\end{aligned}$$

Inform

- $\texttt{Inform}_{S,H}(\varphi)$ means S informs H of φ.
- Assume speaker is honest: precondition is $\boxed{\mathsf{K}_S}\ \varphi$.
- Assume hearer is trusting: effect is $\boxed{\mathsf{K}_H}\ \varphi$.
- Also $\boxed{\mathsf{K}_S}\boxed{\mathsf{K}_H}\ \varphi$, $\boxed{\mathsf{K}_H}\boxed{\mathsf{K}_S}\boxed{\mathsf{K}_H}\ \varphi$, ...
  represent via common knowledge: $\boxed{\mathsf{C}_{\{S,H\}}}\ \varphi$.
- Example: $\texttt{Inform}_{Dog,Crane}(S(\langle 2, 1\rangle))$
  Dog informs Crane that there is stench in square $\langle 2, 1\rangle$.

Inform Action

Can represent as STRIPS-like planning action.

$$\begin{aligned}\texttt{action:}\ & \texttt{Inform}_{S,H}(\varphi)\\ \texttt{precondition:}\ & \boxed{\mathsf{K}_S}\ \varphi\\ \texttt{add:}\ & \boxed{\mathsf{C}_{\{S,H\}}}\ \varphi\\ \texttt{delete:}\ & \end{aligned}$$

Note: assumes all agents have synchronised clocks.

Mid-Lecture Exercise

Suppose there are three agents: A, B, C, and a message φ. Suppose $\boxed{\mathsf{K}_A}\,\varphi$. Can you use $Inform$ to form a plan to achieve $\boxed{\mathsf{C}_{\{A,B,C\}}}\,\varphi$?

Solution to Exercise

No.

It is possible to form plans to achieve all the two-way combinations of common knowledge, i.e. $\boxed{\mathsf{C}_{\{A,B\}}}\,\varphi$, $\boxed{\mathsf{C}_{\{B,C\}}}\,\varphi$ and $\boxed{\mathsf{C}_{\{C,A\}}}\,\varphi$, but not the three-way combination $\boxed{\mathsf{C}_{\{A,B,C\}}}\,\varphi$. For instance, the plan $\texttt{Inform}_{A,B}(\varphi)$, $\texttt{Inform}_{B,C}(\varphi)$ achieves $\boxed{\mathsf{C}_{\{B,C\}}}\,\varphi$. However, the effect of $\texttt{Inform}_{S,H}(\varphi)$ is only to add the two-way common knowledge $\boxed{\mathsf{C}_{\{S,H\}}}\,\varphi$; never a three-way version.

Query

- $\texttt{Query}_{S,H}(\varphi)$ means S asks question φ of H.
- Assume speaker knows that hearer knows the answer.

  Represent as hearer knowing either φ or $\neg\varphi$.
- Assume hearer wants speaker to know this information.

  Use trick that hearer always has goal of making speaker happy.

  Link that happiness to speaker knowing information.

Two Types of Query

Yes/No Query: Asks only to confirm/deny fact.

Yes/No Example: $\texttt{Query}_{Crane,Dog}(S(\langle 2,1\rangle))$.

Crane asks Dog whether there is a stench in square $\langle 2,1\rangle$.

Wh Query: Asks to instantiate a variable.

Who? What? Which? When? How? *etc.*

Wh Example:

$\texttt{Query}_{Crane,Dog}(?sq_2.\ Next(sq_1, dir, sq_2) \wedge OK(sq_2))$.

Crane asks Dog which squares adjacent to sq_1 are safe.

Yes/No Query Action

Can represent as STRIPS-like planning action.

$$\texttt{action:}\texttt{Query}_{S,H}(\varphi)$$
$$\texttt{precondition:}\boxed{\mathsf{K}_S}\ (\boxed{\mathsf{K}_H}\,\varphi \lor \boxed{\mathsf{K}_H}\,\neg\varphi)$$
$$\texttt{effect:}\boxed{\mathsf{K}_H}\ ((\boxed{\mathsf{K}_S}\,\varphi \lor \boxed{\mathsf{K}_S}\,\neg\varphi) \rightarrow \mathrm{happy}(S))$$

Note: effect no longer simple add and delete lists.

Wh Query Action

Can represent as STRIPS-like planning action.

$$\texttt{action:}\texttt{Query}_{S,H}(?x.\varphi(x))$$
$$\texttt{precondition:}\boxed{\mathsf{K}_S}\ \exists x\ \boxed{\mathsf{K}_H}\,\varphi(x)$$
$$\texttt{effect:}\boxed{\mathsf{K}_H}\ \left(\exists x\ \boxed{\mathsf{K}_S}\,\varphi(x) \rightarrow \mathrm{happy}(S)\right)$$

Request

- $\texttt{Request}_{S,H}(\varphi)$ means S asks H to carry out action φ.
- Repeat trick of linking performance of action to happiness of speaker.
- Example: $\texttt{Request}_{Crane,Dog}(Move(Dog, \langle 1,1\rangle, \langle 1,2\rangle))$
  Crane asks Dog to move from square $\langle 1,1\rangle$ to square $\langle 1,2\rangle$.

Request Action

Can represent as STRIPS-like planning action.

$$\texttt{action:}\texttt{Request}_{S,H}(act)$$
$$\texttt{precondition:}$$
$$\texttt{effect:}\boxed{\mathsf{K}_H}\ (\exists i.\ T(act, i) \rightarrow \mathrm{happy}(S))$$

Multi-Agent Plan

- Initial position and heading of Crane given by:

$$At(Crane, \langle 1,1\rangle) \wedge Heading(Crane, West)$$

- Crane asks Dog which adjacent squares are safe:

$$\texttt{Query}_{Crane,Dog}(?sq_2.\ Next(\langle 1,1\rangle, dir, sq_2) \wedge OK(sq_2))$$

- Dog informs Crane of an adjacent square which is safe:

$$\texttt{Inform}_{Dog,Crane}(Next(\langle 1,1\rangle, North, \langle 1,2\rangle) \wedge OK(\langle 1,2\rangle))$$

- Crane then turns in that direction:

$$Right(Crane, West, North)$$

- And moves forward to safe square:

$$Forward(Crane, \langle 1,1\rangle, \langle 1,2\rangle)$$

Modal Inference Required

- Effect of $\texttt{Inform}_{Dog,Crane}(Next(\langle 1,1\rangle, North, \langle 1,2\rangle) \wedge OK(\langle 1,2\rangle))$ is:

$$\boxed{\mathsf{C}_{\{Dog,Crane\}}}\ Next(\langle 1,1\rangle, North, \langle 1,2\rangle) \wedge OK(\langle 1,2\rangle)$$

- But precondition of $Forward(Crane, \langle 1,1\rangle, \langle 1,2\rangle)$requires:

$$Next(\langle 1,1\rangle, North, \langle 1,2\rangle) \wedge OK(\langle 1,2\rangle)$$

- Must use definition
$\boxed{\mathsf{C}_{set}}\ \varphi \Leftrightarrow \boxed{\mathsf{E}_{set}}\ \varphi \wedge \boxed{\mathsf{E}_{set}}\ \boxed{\mathsf{E}_{set}}\ \varphi \wedge \boxed{\mathsf{E}_{set}}\ \boxed{\mathsf{E}_{set}}\ \boxed{\mathsf{E}_{set}}\ \varphi \wedge \ldots$ to infer:

$$\boxed{\mathsf{E}_{\{Dog,Crane\}}}\ Next(\langle 1,1\rangle, North, \langle 1,2\rangle) \wedge OK(\langle 1,2\rangle)$$

- And definition $\boxed{\mathsf{E}_{\{A,B,C,\ldots\}}}\ \varphi \Leftrightarrow \boxed{\mathsf{K}_A}\ \varphi \wedge \boxed{\mathsf{K}_B}\ \varphi \wedge \boxed{\mathsf{K}_C}\ \varphi \wedge \ldots$ to infer:

$$\boxed{\mathsf{K}_{Crane}}\ Next(\langle 1,1\rangle, North, \langle 1,2\rangle) \wedge OK(\langle 1,2\rangle)$$

- Then property **T** ($\boxed{\mathsf{K}_A}\ \varphi \ \rightarrow\ \varphi$) to infer:

$$Next(\langle 1,1\rangle, North, \langle 1,2\rangle) \wedge OK(\langle 1,2\rangle)$$

Conclusion

- Model communication as exchange of modal formulae.
- Use modal logic of knowledge to represent preconditions and effects.
- Three types of communication act: inform, query and request.
- Use plan formation actions to represent each of these.
- Can then combine with regular actions to form multi-agent plans.
- Requires modal inference.