# Logical Agents 2

## Extending the Expressive Power

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(some slides courtesy of Russell and Norvig)



## Disadvantages of Propositional Representation

- ♢ Lots of propositional variables: 112 in Wumpus World.
- ♢ Lots of rules: need to use schemas.
- ♢ Inference space hungry: $2^{112}$ rows in full truth table.

Solution: use FOL representation, where propositions have more internal structure.



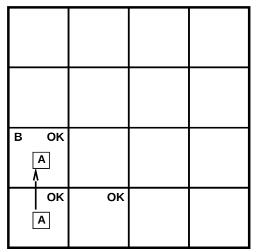
## Representation: Predicates

- ♢ $W(i, j)$ means there is a Wumpus in square $(i, j)$.
- ♢ $S(i, j)$ means there is a stench in square $(i, j)$.
- ♢ $P(i, j)$ means there is a pit in square $(i, j)$.
- ♢ $B(i, j)$ means there is a breeze in square $(i, j)$.
- ♢ $G(i, j)$ means there is gold (and a glitter) in square $(i, j)$.
- ♢ $V(i, j)$ means that square $(i, j)$ has been visited.
- ♢ $OK(i, j)$ means that square $(i, j)$ is safe.

NB – there are only 7 predicates.



## Representation: FOL Knowledge Base




| | | | | | | |
|---|---|---|---|---|---|---|
| $\neg W(1,1)$ | $\neg S(1,1)$ | $\neg P(1,1)$ | $\neg B(1,1)$ | $\neg G(1,1)$ | $V(1,1)$ | $OK(1,1)$ |
| $\neg W(2,1)$ | – | $\neg P(2,1)$ | – | – | $\neg V(2,1)$ | $OK(2,1)$ |
| $\neg W(1,2)$ | $\neg S(1,2)$ | $\neg P(1,2)$ | $B(1,2)$ | $\neg G(1,2)$ | $V(1,2)$ | $OK(1,2)$ |

Facts become known either via sensors as a result of agent actions or via inference using facts and rules.

## Representation: Types of Rules

Diagnostic rule—infer cause from effect

$$\forall i. \forall j.\ S(i,j) \Rightarrow (W(i-1,j) \vee W(i+1,j) \vee W(i,j-1) \vee W(i,j+1))$$

Causal rule—infer effect from cause

$$\forall i. \forall j.\ W(i-1,j) \Rightarrow S(i,j)$$

Neither of these is complete—e.g., the causal rule doesn't say whether squares far away from Wumpuses stench.

Definition for the stench predicate $S$:

$$\forall i. \forall j.\ S(i,j) \Leftrightarrow (W(i-1,j) \vee W(i+1,j) \vee W(i,j-1) \vee W(i,j+1))$$



## Representation: Definitional Rules

◇ A square is safe iff it contains no Wumpus and no pit.

$$\forall i. \forall j.\ OK(i,j) \Leftrightarrow (\neg W(i,j) \wedge \neg P(i,j))$$

◇ A stench iff a Wumpus in an adjacent square.

$$\forall i. \forall j.\ S(i,j) \Leftrightarrow (W(i-1,j) \vee W(i+1,j) \vee W(i,j-1) \vee W(i,j+1))$$

◇ A breeze iff a pit in an adjacent square.

$$\forall i. \forall j.\ B(i,j) \Leftrightarrow (P(i-1,j) \vee P(i+1,j) \vee P(i,j-1) \vee P(i,j+1))$$

NB – define predicates over non-existent squares to be false, $e.g.\ \neg W(0,1),\ \neg P(2,5)$.

Unwanted disjuncts are simplified away:

$$\begin{aligned} S(1,1) &\Leftrightarrow W(0,1) \vee W(2,1) \vee W(1,0) \vee W(1,2) \\ &\Leftrightarrow W(2,1) \vee W(1,2) \end{aligned}$$



## Putting Definitions in Clausal Form

◇ Original Definition:

$$\forall i. \forall j.\ S(i,j) \Leftrightarrow (W(i-1,j) \vee W(i+1,j) \vee W(i,j-1) \vee W(i,j+1))$$

◇ Skolemize:

$$S(i,j) \Leftrightarrow (W(i-1,j) \vee W(i+1,j) \vee W(i,j-1) \vee W(i,j+1))$$

◇ Split into Two Implications:

$$S(i,j) \Rightarrow (W(i-1,j) \vee W(i+1,j) \vee W(i,j-1) \vee W(i,j+1))$$
$$(W(i-1,j) \vee W(i+1,j) \vee W(i,j-1) \vee W(i,j+1)) \Rightarrow S(i,j)$$

◇ Split Second Implication into Clauses:

$$\begin{aligned} W(i-1,j) &\Rightarrow S(i,j) \\ W(i+1,j) &\Rightarrow S(i,j) \\ W(i,j-1) &\Rightarrow S(i,j) \\ W(i,j+1) &\Rightarrow S(i,j) \end{aligned}$$



## Inference Using Resolution

◇ Clauses:

$$W(i-1,j) \Rightarrow S(i,j) \quad (1)$$
$$W(i+1,j) \Rightarrow S(i,j) \quad (2)$$
$$W(i,j-1) \Rightarrow S(i,j) \quad (3)$$
$$W(i,j+1) \Rightarrow S(i,j) \quad (4)$$

◇ KB: (5) $S(1,2) \Rightarrow$, (6) $W(1,1) \Rightarrow$, (7) $W(0,2) \Rightarrow$, ...

◇ Inference 1: Resolve (5) and (2) to get: $W(2,2) \Rightarrow$.

◇ Inference 2: Resolve (5) and (4) to get: $W(1,3) \Rightarrow$.

◇ Conclusion: Add both $\neg W(2,2)$ and $\neg W(1,3)$ to KB.

NB – Assume arithmetic evaluation is built-in, $e.g.\ 1+1$ evaluates to $2$.

## Mid-Lecture Exercise

◊ Clauses:

(0) $B(i,j) \Rightarrow (P(i-1,j) \vee P(i+1,j) \vee P(i,j-1) \vee P(i,j+1))$
(2) $P(i-1,j) \Rightarrow B(i,j)$
(3) $P(i+1,j) \Rightarrow B(i,j)$
(4) $P(i,j-1) \Rightarrow B(i,j)$
(5) $P(i,j+1) \Rightarrow B(i,j)$

◊ KB: $(6) \Rightarrow B(2,2), (7)P(1,2) \Rightarrow, (8)P(2,1) \Rightarrow, (9)P(2,3) \Rightarrow$

◊ Goal: $(10)P(3,2) \Rightarrow$



## Answer to Mid-Lecture Exercise

◊ Clauses:

$(0)B(i,j) \Rightarrow (P(i-1,j) \vee P(i+1,j) \vee P(i,j-1) \vee P(i,j+1))$

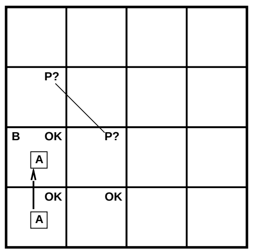
◊ KB: $(6) \Rightarrow B(2,2)$, $(7)P(1,2) \Rightarrow$, $(8)P(2,1) \Rightarrow$, $(9)P(2,3) \Rightarrow$

◊ Goal: $(10)P(3,2) \Rightarrow$

Resolve (6) and (0): $(11) \Rightarrow P(1,2) \vee P(3,2) \vee P(2,1) \vee P(2,3)$
Resolve (11) and (10): $(12) \Rightarrow P(1,2) \vee P(2,1) \vee P(2,3)$
Resolve (12) and (7): $(13) \Rightarrow P(2,1) \vee P(2,3)$
Resolve (13) and (8): $(14) \Rightarrow P(2,3)$
Resolve (14) and (9): $(15) \Rightarrow$



## More Inference: Where is the Pit?

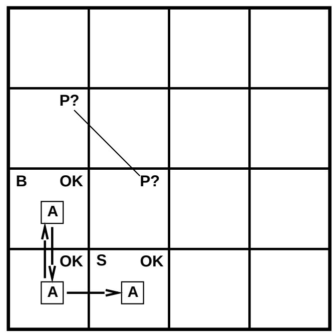



Clause: (1) $B(i,j) \Rightarrow (P(i-1,j) \vee P(i+1,j) \vee P(i,j-1) \vee P(i,j+1))$
KB: $(2) \Rightarrow B(1,2)$, $(3)\ P(1,1) \Rightarrow$, $(4)\ P(0,2) \Rightarrow$
Inference: Resolve (2) and (1) to get: $(5) \Rightarrow P(0,2) \vee P(2,2) \vee P(1,1) \vee P(1,3)$
Resolve (3) and (5) to get: $(6) \Rightarrow P(0,2) \vee P(2,2) \vee P(1,3)$
Resolve (4) and (6) to get: $(7) \Rightarrow P(2,2) \vee P(1,3)$



## Updating a FOL KBs

Suppose a Wumpus-world agent is using an FOL KB
and perceives a stench but no breeze or glitter in square (2,1):




$\text{TELL}(KB, S(2,1))$
$\text{TELL}(KB, \neg B(2,1))$
$\text{TELL}(KB, \neg G(2,1))$

## Interrogating a FOL KBs

The agent may ask whether square (3,1) is ok.

$\textsc{Ask}(KB, OK(3,1))$

Answer: No. In fact, $OK(3,1) \Rightarrow$ (since $W(3,1)$)

$$
\begin{array}{ll}
(0) & \Rightarrow OK(3,1) \\
(1) & W(i,j) \wedge OK(i,j) \Rightarrow \\
(2) & S(i,j) \Rightarrow W(i-1,j) \vee W(i+1,j) \vee W(i,j-1) \vee W(i,j+1) \\
(3) & \Rightarrow S(2,1) \\
(4) & W(1,1) \Rightarrow \quad (5) \quad W(2,0) \Rightarrow \quad (6) \quad W(2,2) \Rightarrow \\
(7) & \Rightarrow W(3,1) \quad \text{by (2)-(6)} \\
(8) & \Rightarrow \quad \text{by (7), (1) \& (0)}
\end{array}
$$



## A More Succinct Representation

- Represent squares as pairs $< i, j >$.
- Make predicates unary, *e.g.* $W(< i, j >)$, $S(< i, j >)$.
- Introduce binary adjacency predicate $Adj(< 1, 2 >, < 2, 2 >), \ldots$.
- Avoid disjunctions in rules:

$$\forall p.\ S(p) \Leftrightarrow \exists q.\ Adj(p,q) \wedge W(q)$$

- Avoids need for arithmetic and special cases.



## Clauses for New Representation

**Diagnostic Rules:**

$$S(p) \Rightarrow Adj(p, a(p))$$
$$S(p) \Rightarrow W(a(p))$$

**Causal Rule:**

$$W(q) \wedge Adj(p,q) \Rightarrow S(p)$$



## Representing Time

- In Wumpus World some facts change as the game progresses, *e.g.* whether square has been visited.
- Those predicates whose meaning may change are called *fluents*.
- They are given an extra argument representing time, *e.g.* $V(p,t)$.
- The time argument may be expressed in many different ways. For now, we will use integers, *e.g.* $\neg V(< 1, 2 >, 0)$, $V(< 1, 2 >, 1)$.

## Representing Actions

- Need ability to move in Wumpus World.
  Introduce move predicate: $Move(p, q, t)$.
- And position predicate: $At(p, t)$.
- Need new rules to describe these:

$$At(p,t) \land Adj(p,q) \land Move(p,q,t) \Rightarrow At(q,t+1)$$
$$At(p,t) \land t' \geq t \Rightarrow V(p,t')$$



## Conclusion

- FOL gives a more succinct representation than propositional logic.
- Knowledge base consists of facts and rules.
- Putting definitions in clausal form produces causal and diagnostic rules.
- Represent time by additional argument.
- Use resolution to infer new information from old.
- Can represent actions, but need to decide what actions to perform.