# AI2 Module 3
# Tutorial 5: Sample Solutions

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The main aim for this tutorial is to get some practice with computing using Bayes Nets. It is important that you become familiar with table manipulations and know where to look for what.

During computation, it is important that you write down and then manipulate the formulae for as long as possible, with substitution for actual values (where appropriate) only done as the final step.

**Part 1**

1. The table with have $2^2 3^2 = 72$ rows and 5 columns.

2. We want to compute $\texttt{Pr}\{D = 1|C = a, P = 0, W = 1\}$.
   $\texttt{Pr}\{D = 1|C = a, P = 0, W = 1\} =$
   $\sum_v \texttt{Pr}\{D = 1|C = a, P = 0, W = 1, S = v\}\texttt{Pr}\{S = v|C = a, P = 0, W = 1\} =$
   $\sum_v \texttt{Pr}\{D = 1|C = a, S = v\}\texttt{Pr}\{S = v|P = 0, W = 1\} =$
   $(0.9, 0.5, 0.7) \cdot (0.1, 0.8, 0.1) = 0.56$

3. We need to sum out the effect of $W$ so
   $\mathbf{Pr}(S|P = 0) = 0.3 \cdot (0.8, 0.1, 0.1) + 0.7 \cdot (0.1, 0.8, 0.1) = (0.31, 0.59, 0.1)$
   $\mathbf{Pr}(S|P = 1) = 0.3 \cdot (0, 0.8, 0.2) + 0.7 \cdot (0, 0.7, 0.3) = (0, 0.73, 0.27)$

4. Should sum out the effect of $C$:
   $\mathbf{Pr}(D = 0|S) = \sum_v \texttt{Pr}\{C = v\}\mathbf{Pr}(D = 0|C = v \wedge S) =$
   $0.35 \cdot (0.1, 0.5, 0.3) + 0.3 \cdot (0.7, 0.5, 0.3) + 0.35 \cdot (0.7, 0.5, 0.3) = (0.49, 0.5, 0.7)$
   and the complement is $\mathbf{Pr}(D = 1|S) = (0.51, 0.5, 0.3)$

   NB the above is parameterised by $(s, m, l)$.

   An alternative way of performing these computations is given below:

   $\mathbf{Pr}(D|S = s) = \sum_v \texttt{Pr}\{C = v\}\mathbf{Pr}(D|C = v \wedge S = s) =$
   $0.35 \cdot (0.1, 0, 9) + 0.3 \cdot (0.7, 0.3) + 0.35 \cdot (0.7, 0.3) = (0.49, 0.51)$

   $\mathbf{Pr}(D|S = m) = 0.35 \cdot (0.5, 0, 5) + 0.3 \cdot (0.5, 0.5) + 0.35 \cdot (0.5, 0.5) = (0.5, 0.5)$

   $\mathbf{Pr}(D|S = l) = 0.35 \cdot (0.3, 0, 7) + 0.3 \cdot (0.3, 0.7) + 0.35 \cdot (0.3, 0.7) = (0.3, 0.7)$

5. The first thing to note is that the formula given can be simplified since $D$ is independent of $P$ given $S$:
   $\texttt{Pr}\{D = v_1|P = v_2\} =$
   $\sum_{v_3} \texttt{Pr}\{S = v_3|P = v_2\}\texttt{Pr}\{D = v_1|P = v_2, S = v_3\}$
   $\sum_{v_3} \texttt{Pr}\{S = v_3|P = v_2\}\texttt{Pr}\{D = v_1|S = v_3\}$
   The rest is just combining the tables that we computed before:
   $\texttt{Pr}\{D = 0|P\} = ((0.31, 0.59, 0.1) \cdot (0.49, 0.5, 0.3), (0, 0.73, 0.27) \cdot (0.49, 0.5, 0.3)) = (0.4769, 0.446)$
   we can take the complement directly: $\texttt{Pr}\{D = 1|P\} = (0.5231, 0.554)$.

   Note that the equations given are compactly parametrised by $P = (0, 1)$ but you can run the expanded version just as well.

6. You should recall Bayes Rule with an extra conditioning (just take out $C = a$ and you get the standard formula).

$\mathtt{Pr}\{P = 1|D = 1 \wedge C = a\} = \frac{\mathtt{Pr}\{D=1|P=1\wedge C=a\}\mathtt{Pr}\{P=1|C=a\}}{\mathtt{Pr}\{D=1|C=a\}} =$
$\frac{\mathtt{Pr}\{D=1|P=1\wedge C=a\}\mathtt{Pr}\{P=1\}}{\mathtt{Pr}\{D=1|C=a\}} =$
$\frac{\alpha}{\mathtt{Pr}\{D=1|C=a\}} = \frac{0.554\cdot 0.5}{\mathtt{Pr}\{D=1|C=a\}} = \frac{0.277}{\mathtt{Pr}\{D=1\}}$
$\mathtt{Pr}\{P = 0|C = a \wedge D = 1\} = \frac{\mathtt{Pr}\{D=1|P=0\wedge C=a\}\mathtt{Pr}\{P=0|C=a\}}{\mathtt{Pr}\{D=1|C=a\}} =$
$\frac{\mathtt{Pr}\{D=1|P=0\wedge C=a\}\mathtt{Pr}\{P=0\}}{\mathtt{Pr}\{D=1|C=a\}} =$
$= \frac{\beta}{\mathtt{Pr}\{D=1|C=a\}} = \frac{0.644\cdot 0.5}{\mathtt{Pr}\{D=1|C=a\}} = \frac{0.322}{\mathtt{Pr}\{D=1\}}$

Where the first element is computed as in the previous part (taking account of extra evidence):
$\mathtt{Pr}\{D = 1|P \wedge C = a\} =$
$\sum_v \mathtt{Pr}\{S = v|P \wedge C = a\}\mathtt{Pr}\{D = 1|P \wedge C = a \wedge S = v\} =$
$\sum_v \mathtt{Pr}\{S = v|P\}\mathtt{Pr}\{D = 1|C = a \wedge S = v\} =$
$((0.31, 0.59, 0.1) \cdot (0.9, 0.5, 0.7), (0, 0.73, 0.27) \cdot (0.9, 0.5, 0.7)) = (0.644, 0.554)$

Finally, instead of computing $\mathtt{Pr}\{D = 1|C = a\}$ directly we normalise the two probabilities. $\mathtt{Pr}\{P = 1|C = a \wedge D = 1\} = \alpha/(\alpha + \beta) = 0.462$.

7. In likelihood weighting simulation we fix the evidence values but multiply the weight by the probability to get them given their parents. The query variable and other variables are drawn randomly. We count the round towards “success” if the query variable is assigned the requested value. The estimate is the “success count” divided by the “total count”.

   We are computing $\mathtt{Pr}\{P = 1|C = a \wedge D = 1\}$ and values $(C, P, W, S, D) = (a, 1, 0, s, 1)$ were drawn (assume from left to right in order).

   The computation is as follows:
   Initialise $w = 1$.
   $C = a$ so we update $w = w \cdot \mathtt{Pr}\{C = a\} = 1 \cdot 0.35 = 0.35$.
   $P$ is drawn randomly.
   $W$ is drawn randomly.
   $S$ is drawn randomly.
   $D = 1$ so we update $w = w \cdot \mathtt{Pr}\{D = 1|C = a, P = 1, W = 0, S = s\} = 0.35 \cdot 0.9 = 0.315$
   $w = 0.315$ is the final weight.

   Since $P = 1$ was obtained both “success count” and “total count” are increased by 0.315.

**Part 2**

CPTs available are: $\mathbf{Pr}(C|A,B)$, $\mathbf{Pr}(B|A)$, $\mathbf{Pr}(A)$, $\mathbf{Pr}(D|C)$. We can derive the formula as follows:
$\mathbf{Pr}(D|A) = \sum_v \mathtt{Pr}\{D, C = v|A\}$
by definition of marginal distribution;
$= \sum_v \mathtt{Pr}\{D|C = v, A\}\mathtt{Pr}\{C = v|A\}$
$= \sum_v \mathtt{Pr}\{D|C = v\}\mathtt{Pr}\{C = v|A\}$
since $D$ is independent of $A$ given $C$;
$= \sum_v \mathtt{Pr}\{D|C = v\} \sum_w \mathtt{Pr}\{C = v, B = w|A\}$
by definition of marginal distribution;
$= \sum_v \mathtt{Pr}\{D|C = v\} \sum_w \mathtt{Pr}\{C = v|A, B = w\}\mathtt{Pr}\{B = w|A\}$