AI2 Module 3
Tutorial 5

Jacques Fleuriot
School of Informatics
(Amended by David Talbot February 3, 2005.)

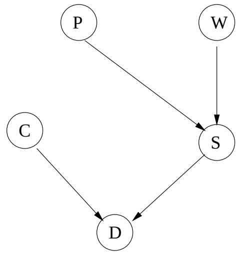
In this tutorial we revise computations with Bayesian Networks. If you do not have enough time to complete all computations in class please continue on your own so as to make sure you are comfortable with the various notions.

Part 1

Encouraged by the previous study (of tutorial 4) the management committee of the carnival in Brazil decided to gather further statistics on participants in the performances. The scheme this year prescribed that each person who wanted to participate put forward an application through one of the many dance schools in the country. Applications were then examined individually and a decision made in each case.

In the current study 5 properties were recorded for each applicant:

- P is a Boolean variable recording whether the applicant participated in the carnival in the previous year.
- S is the size of the school through which the application was put forward; possible values recorded were small, medium, large, denoted below as s, m, l.
- D is the decision, 1 for accepted and 0 otherwise.
- W is a Boolean variable recording whether the applicant took a preparation workshop in the school before applying.
- C records the home city of the applicant (with values: a, b, c).



Since the joint distribution table is not small it was decided to use a Bayesian Network to represent it. The distribution over these variables was analysed and found to conform with the network drawn on the right. The conditional distribution tables are as follows.

$\mathtt{Pr}\{W=0\}$	$\mathtt{Pr}\{W=1\}$
0.3	0.7

$\mathtt{Pr}\{P=0\}$	$\mathtt{Pr}\{P=1\}$
0.5	0.5

$\mathtt{Pr}\{C=a\}$	$\mathtt{Pr}\{C=b\}$	$\mathtt{Pr}\{C=c\}$
0.35	0.3	0.35

P	W	$\mathtt{Pr}\{S=s\}$	$\mathtt{Pr}\{S=m\}$	$\mathtt{Pr}\{S=l\}$
0	0	0.8	0.1	0.1
0	1	0.1	0.8	0.1
1	0	0	0.8	0.2
1	1	0	0.7	0.3

C	S	$\mathtt{Pr}\{D=0\}$	$\mathtt{Pr}\{D=1\}$
a	s	0.1	0.9
a	m	0.5	0.5
a	l	0.3	0.7
b	s	0.7	0.3
b	m	0.5	0.5
b	l	0.3	0.7
c	s	0.7	0.3
c	m	0.5	0.5
c	l	0.3	0.7

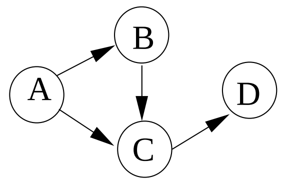
Questions: Recall from the slides that $\mathtt{Pr}\{\}$ denotes the probability of a particular event and $\mathbf{Pr}()$ denotes the marginal distribution over a variable or a set of variables.

1. What is the size of the table needed to describe the joint distribution directly ?
2. What is the probability that a new applicant ($P = 0$) from city a ($C = a$) who took the preparation workshop ($W = 1$) is accepted ($D = 1$)?
3. Compute a table for $\mathbf{Pr}(S|P)$.
4. Compute a table for $\mathbf{Pr}(D|S)$.
5. Use the previous two parts to compute a table for $\mathbf{Pr}(D|P)$.
   You may want to use the following formula as the basis for the computation:
   $\mathtt{Pr}\{D=v_1|P=v_2\} = \sum_{v_3} \mathtt{Pr}\{S=v_3|P=v_2\}\mathtt{Pr}\{D=v_1|P=v_2, S=v_3\}$
6. What is the probability that an applicant from city a who was accepted, participated in the carnival in the previous year ? (Compute $\mathtt{Pr}\{P=1|C=a \wedge D=1\}$)

   You can use Bayes' Rule as in the formula below and use normalisation by computing similar value with $P = 0$ as well.
   $\mathtt{Pr}\{P=1|D=1 \wedge C=a\} = \frac{\mathtt{Pr}\{D=1|P=1 \wedge C=a\}\mathtt{Pr}\{P=1|C=a\}}{\mathtt{Pr}\{D=1|C=a\}}$
7. Instead of performing the previous computation for $\mathtt{Pr}\{P=1|C=a \wedge D=1\}$ one can try to estimate the probability using the likelihood weighting simulation. Imagine that in one round of the simulation a sample with values $(C, P, W, S, D) = (a, 1, 0, s, 1)$ is drawn. What weight is assigned to this round ? and how does the sample contribute to the estimate ?

Part 2

Questions (from June 2002 Exam). Consider the following belief network:



Derive a formula for $\mathbf{Pr}(D|A)$ in terms of some or all of the following conditional probability distributions: $\mathbf{Pr}(C|A, B)$, $\mathbf{Pr}(B|A)$, $\mathbf{Pr}(D|C)$, $\mathbf{Pr}(A)$.