AI2 Module 3
Tutorial 4: Sample Solutions

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The main aim for this tutorial is to get some practice with the various notions and with computations using probabilities. As a side effect you should realise that computations are time consuming so that using structure in Bayes nets is advantageous.

Part 1

In this part, you should know what to compute (which entries to sum) or compare (for independence).

1. $\mathtt{Pr}\{D = 1|P = 0\} = \frac{\mathtt{Pr}\{D=1 \wedge P=0\}}{\mathtt{Pr}\{P=0\}} = \frac{0.26155}{0.5} = 0.5231$

   For $\mathtt{Pr}\{D = 1 \wedge P = 0\}$ sum over values of S to get: $0.07905 + 0.1475 + 0.035 = 0.26155$.

   For $\mathtt{Pr}\{P = 0\}$ sum over two other variables to get: 0.5

2. For $\mathbf{Pr}(D)$ we sum over two other variables for each value of D. This gives
   $\mathtt{Pr}\{D = 0\} = 0.07595 + 0.1475 + 0.015 + 0 + 0.1825 + 0.0405 = 0.46145$,
   and therefore $\mathtt{Pr}\{D = 1\} = 0.53855$.

   To summarise (order values as (0,1)): $\mathbf{Pr}(D) = (0.46145, 0.53855)$

3. $\mathtt{Pr}\{D = 1|P = 0\} \neq \mathtt{Pr}\{D = 1\}$ so they are not independent.

4. No, as $\mathbf{Pr}(D|P) = \mathbf{Pr}(D)$ means that $\mathbf{Pr}(D|P = v) = \mathbf{Pr}(D)$ for any v and this is false by the previous part.

5. $\mathbf{Pr}(D|S = l) = \frac{\mathbf{Pr}(D, S=l)}{\mathtt{Pr}\{S=l\}}$
   $\mathtt{Pr}\{D = 0 \wedge S = l\} = 0.015 + 0.0405 = 0.0555$
   $\mathtt{Pr}\{D = 1 \wedge S = l\} = 0.035 + 0.0945 = 0.1295$
   $\mathtt{Pr}\{S = l\} = 0.185$

   Finally, we get $\mathtt{Pr}\{D|S = l\} = (0.0555/0.185, 0.1295/0.185) = (0.3, 0.7)$

6. Since $\mathtt{Pr}\{S = l \wedge P = 0\} = 0.035 + 0.015 = 0.05$
   $\mathtt{Pr}\{D = 1|S = l \wedge P = 0\} = \frac{\mathtt{Pr}\{D=1 \wedge S=l \wedge P=0\}}{\mathtt{Pr}\{S=l \wedge P=0\}} = \frac{0.035}{0.05} = 0.7$
   and $\mathbf{Pr}(D|S = l \wedge P = 0) = (0.3, 0.7)$

   Since $\mathtt{Pr}\{S = l \wedge P = 1\} = 0.0405 + 0.0945 = 0.135$
   $\mathtt{Pr}\{D = 1|S = l \wedge P = 1\} = \frac{\mathtt{Pr}\{D=1 \wedge S=l \wedge P=1\}}{\mathtt{Pr}\{S=l \wedge P=1\}} = \frac{0.0945}{0.135} = 0.7$
   and $\mathbf{Pr}(D|S = l \wedge P = 1) = (0.3, 0.7)$

7. From 5 and 6: $\mathbf{Pr}(D|S = l \wedge P = 0) = \mathbf{Pr}(D|S = l \wedge P = 1) = \mathbf{Pr}(D|S = l)$
   and therefore D is independent of P given $S = l$.

8. Will need to verify the same for $S = m$ and $S = s$.

Part 2

1. $\texttt{Pr}\{X|Y\} = \frac{\texttt{Pr}\{X \cap Y\}}{\texttt{Pr}\{Y\}}$

2. Proof that $\texttt{Pr}\{X, Y|Z\} = \texttt{Pr}\{X|Y, Z\}\texttt{Pr}\{Y|Z\}$:

$$\texttt{Pr}\{X|Y, Z\}\texttt{Pr}\{Y|Z\} = \frac{\texttt{Pr}\{X, Y, Z\}}{\texttt{Pr}\{Y, Z\}} \cdot \frac{\texttt{Pr}\{Y, Z\}}{\texttt{Pr}\{Z\}} = \frac{\texttt{Pr}\{X, Y, Z\}}{\texttt{Pr}\{Z\}}$$

$$= \frac{\texttt{Pr}\{X, Y|Z\}\texttt{Pr}\{Z\}}{\texttt{Pr}\{Z\}} = \texttt{Pr}\{X, Y|Z\}$$