AI2 Module 3
Tutorial 4

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In this tutorial we review basic computations in product probability spaces. Please try to complete all the computations to make sure you are comfortable with the various concepts.

Part 1

The management committee of the carnival in Brazil decided to gather statistics on participants in the performances. The scheme this year prescribed that each person who wanted to participate put forward an application through one of the many dance schools in the country. Applications were then examined individually and a decision made in each case.

A pilot study was started by recording 3 properties for each applicant:

- P is a Boolean variable recording whether the applicant participated in the carnival in the previous year.
- S is the size of the school through which the application was put forward; possible values recorded were small, medium, large, denoted below as s, m, l.
- D is the decision, 1 for accepted and 0 otherwise.

The distribution over applicants is summarised in the table.

P	S	D	Pr{}
0	s	0	0.07595
0	s	1	0.07905
0	m	0	0.1475
0	m	1	0.1475
0	l	0	0.015
0	l	1	0.035
1	s	0	0.0
1	s	1	0.0
1	m	0	0.1825
1	m	1	0.1825
1	l	0	0.0405
1	l	1	0.0945

Questions: Recall from the slides that Pr{} denotes the probability of a particular event and **Pr**() denotes the marginal distribution over a variable or a set of variables.

1. Compute $\text{Pr}\{D = 1|P = 0\}$
   (the probability that a new applicant is accepted)
2. Compute $\mathbf{Pr}(D)$
3. Is the event $D = 1$ "the applicant is accepted" independent of the event $P = 0$ "a new applicant" ?
4. Is D independent of P ?
5. Compute $\mathbf{Pr}(D|S = l)$.
6. Compute $\mathbf{Pr}(D|S = l, P)$.
   (For each value v of P, compute the conditional distribution $\mathbf{Pr}(D|S = l, P = v)$.)
7. Verify that D is conditionally independent of P given $S = l$.
8. In fact, for the distribution we have, D is conditionally independent of P given S. What would you need to check in order to verify that ?

Part 2

Questions (from June 2001)

1. State the conditional probability rule.
2. Using your answer above, or otherwise, prove that

$$\text{Pr}\{X, Y|Z\} = \text{Pr}\{X|Y, Z\}\text{Pr}\{Y|Z\}$$