AI2 Module 3: Assignment (Part B)

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This document describes Part B of the assignment for AI2 Module 3. Written or typed answers to this assignment should be submitted to the **Informatics Teaching Office** by **5 pm, Friday, 18th February 2005**. Late submissions will in general not be accepted — see Students' guide to AI2.

If you have any problems, queries or are just stuck please feel free to email me at `d.r.talbot@sms.ed.ac.uk` or find me in Floor 3, Rm. 9, 2 Buccleuch Place *during my office hours*; Tuesday 2–4 and Thursday 11–12.

1 Probability and Baysian Networks

In this assignment you are asked to derive various formula, calculate probabilities and construct a Baysian Network. **A large portion of the marks will be give for clear explanation of each and every step involved in any derivation or calculation.**

Recall that $\texttt{Pr}\{\}$ denotes the probability of a particular event and $\mathbf{Pr}()$ denotes the marginal distribution over a variable or a set of variables.

1.1 Conditional Probability (20%)

A group of friends are playing the South America dice game Perudo. Each player having shaken their cup of dice turns it over and looks at what they have rolled but without revealing their dice to the other players. One of the players who has four dice left, accidentally remarks out loud that the values he has rolled add up to an even number. In order to judge what they should bid the players that heard this remark try and calculate the probability that the player has four dice showing the same value.

*Based on material by Paul Crook.

To keep things compact we'll use the letter E to designate the occurrence of the event that the face values of four dice added up to an even number, and I to designate the event that four dice are displaying identical values.

(a) Write down using conditional probability notation the single, simple expression which captures the following phrase: *The probability that the face values shown by all four dice are identical given that the total face value of the same dice add up to an even number.*

(b) The single expression in answer to (a) above can be expressed in terms of the probability of both events occurring together, divided by the probability of one of the events occurring by itself. Re-express the answer to (a) in this form and show how this form can be derived. Explain clearly each step of you derivation and any notation that you introduce. The majority of marks will be given for explanation which demonstrates understanding of the derivation, not for just reproducing the derivation.

(c) Assuming four, six sided, fair dice calculate the values of $\texttt{Pr}\{E\}$ and $\texttt{Pr}\{I\}$, explaining how you arrived at your answer.

(d) Based on the above two answers can you *easily* calculate the probability of both event occurring at the same time, *i.e.* $\texttt{Pr}\{E \cap I\}$ — think carefully about this. If it is not easily possible then explain why.

(e) You could attempt to answer the question that the above players are trying to solve by constructing a joint distribution table, *i.e.* a table that list every possible combination of events and the probability of each combination occurring. Explain why you would not want to construct the joint distribution table for rolling four dice.

1.2 Bayes' Theorem (20%)

(a) State the Conditional Probability rule (i.e. give a definition of conditional probability).

(b) State Bayes' Theorem.

(c) Show how Bayes' Theorem can be derived from the Conditional Probability rule. Explain clearly each step of your derivation and any notation that you introduce. The majority of marks will be given for explanation which demonstrates understanding of the derivation, not for just reproducing the derivation.

(d) Using Bayes' Theorem, and explaining clearly how you calculated it, answer the question that the players in the above scenario are trying to solve. That is, calculate the probability that the face values shown by four dice are identical given that the total face values of all the dice add up to an even number.

1.3 Marginal Distributions (20%)

The probability of being able to make a call, variable C, on a mobile phone is dependent on three independent variables:

- Is the battery okay ($B = okay$) or flat ($B = flat$).
- Is the signal good ($S = good$), poor ($S = poor$) or non existent ($S = none$).
- Is the network busy ($N = busy$) or ($N = free$).

Where $C = 1$ means it is possible to make a call and $C = 0$ it isn't possible to call.

B	S	N	C	Pr{}
okay	good	free	0	0.0136
okay	good	free	1	0.2579
okay	good	busy	0	0.0143
okay	good	busy	1	0
okay	poor	free	0	0.2036
okay	poor	free	1	0.2036
okay	poor	busy	0	0.0214
okay	poor	busy	1	0
okay	none	free	0	0.1357
okay	none	free	1	0
okay	none	busy	0	0.0071
okay	none	busy	1	0
flat	good	free	0	0.0452
flat	good	free	1	0
flat	good	busy	0	0.0024
flat	good	busy	1	0
flat	poor	free	0	0.0678
flat	poor	free	1	0
flat	poor	busy	0	0.0036
flat	poor	busy	1	0
flat	none	free	0	0.0226
flat	none	free	1	0
flat	none	busy	0	0.0012
flat	none	busy	1	0

Table 1: Joint distribution table for making a call on a fictitious mobile phone.

Given the joint distribution table shown in the table above calculate the following *marginal distribution tables*, clearly setting out your working:

(a) Probability distribution of the state of the battery, $\mathbf{Pr}(B)$;

(b) Probability distribution of the signal, $\mathbf{Pr}(S)$;

(c) Probability distribution of being able to make a call, $\mathbf{Pr}(C)$;

(d) Probability distribution for $\mathbf{Pr}(C \cap B)$;

(e) Probability distribution of being able to make a call conditioned by the state of the battery, $\mathbf{Pr}(C|B)$.

1.4 Bayesian Networks (40%)

In this section you are asked to construct a Bayesian network. The network is based on the following scenario:

> A fictitious student is enrolled in a course for which there is a 9am lecture (variable L) on Friday morning. Sometimes she attends this lecture ($L = a$) other times she misses it ($L = m$).
>
> The chances of her attending the lecture depends in part on what she did on Thursday night (variable N), stayed in ($N = s$), went to the pub ($N = p$) or had a big night out ($N = n$).
>
> There is also a tutorial on Tuesday afternoon (variable T). Independent of her attendance of the lecture, or what she did the night before, she may attend this tutorial ($T = a$) or not ($T = m$).
>
> There is a small chance that she might go away on a holiday for a week (variable $H = 1$) or not ($H = 0$). If she does go on holiday this will effect her attendance at both the lecture and tutorial. Assume that the probability of going on a Thursday night out is independent of being on holiday, its just the location that changes.

(a) Construct a Bayesian network whose structure represents the dependencies (and independence) in the scenario above.

(b) Based on the information above, or otherwise your intuition, assign conditional probability tables for all nodes in the network. Briefly justify your choices.

(c) Express the joint distribution $\mathbf{Pr}(L, N, T, H)$ as the product of the distributions in your Bayesian network (as per lecture slides 3-7 & 3-8).

(d) Clearly setting out the steps of your calculation and only plugging in numbers as the last step to produce a result, use the Bayesian network you constructed to compute:

(i) A table for $\mathbf{Pr}(L|N)$

(ii) The probability of the student missing a lecture, $\texttt{Pr}\{L = m\}$

(iii) The probability of the student missing a tutorial, $\texttt{Pr}\{T = m\}$

(iv) If the student was seen in a night club on Thursday night what is the probability of her attending the lecture, i.e. what is $\texttt{Pr}\{L = a|N = n\}$.

(v) Probability of the student being on holiday if it is observed that she missed the lecture.

(vi) Probability of the student being on holiday if it is observed that she missed the tutorial.

(vii) Probability of the student being on holiday if it is observed that she misses both the tutorial and lecture.

(e) Comment on the results of questions (v), (vi) and (vii). Do they seem reasonable?

(f) By reference to your network diagram or otherwise, comment on the relationship between the lecture (L) and tutorial (T) when holiday (H) is observed.

Notes:

- In constructing your Bayesian Network you need only consider the dependencies that are explicitly expressed in the scenario. If no linkage is explicitly mentioned between two variables you can assume they are independent of each other (even if intuitively you might want to link them).
- In all parts above you should show your computations by using appropriate formulas and justifying derivation steps. Only then plug in numbers to produce a result. A significant part of the marks will be assigned to clear and correct presentation of formulas and justification of derivation steps.
- The scenario described implies that some of the probabilities in the tables are zero or one. This should help you simplify the actual computations.
- In some of the parts you may be able to use values already computed in previous parts.

2 Assignments and Plagiarism

Some comments about plagiarism:

1. Submitting another student's script, even if modified, as if it is your original work is plagiarism.

2. Assisting another student to plagiarise (e.g. by sharing scripts) is also penalised.
3. Discussing the assignment in broad terms is okay, but NOT at the level of derivations and computational details.
4. Even partial, incomplete submissions get partial credit.
5. A failed assignment is not a failed career. Getting caught at plagiarism could ruin it.
6. We use various techniques to detect plagiarism.
7. If you can't do the assignment, discuss this with the course organiser or director studies. Remember that the Demonstrator (Teaching Assistant) is there to help you throughout each assignment (not just a few days before the deadline).
8. Change the protections on your work directory and files so potential plagiarists cannot access your solution.