AGTA Tutorial 7

Please attempt the question before your tutorial.

1. Consider the 6-state MDP depicted in Figure 1.

In this MDP, states $s_5$ and $s_3$ are controlled by the player (the controller), whereas the other states are controlled by “nature” (random).

Suppose the player’s objective is to optimize the probability of reaching state $s_6$ starting at each state $s_i$, $i = 1, \ldots, 5$.

Write the Bellman optimality equations for this MDP with this objective.

Compute the optimal probabilities, $p^*_i$, of reaching state $s_6$ starting from state $s_i$, for $i = 1, \ldots, 5$, and also compute an optimal strategy for the player.

2. Consider the atomic network congestion game, with three players, described by the directed graph in Figure 2.

In this game, every player $i$ (for $i = 1, 2, 3$) needs to choose a directed path from the source $s$ to the target $t$. Thus, every player $i$’s set of possible actions (i.e., its set of pure strategies) is the set of all possible directed paths from $s$ to $t$.

Each edge $e$ is labeled with a sequence of three numbers $(c_1, c_2, c_3)$. Given a profile $\pi = (\pi_1, \pi_2, \pi_3)$ of pure strategies (i.e., $s$-$t$-paths) for all three players, the cost to player $i$ of each directed edge, $e$, that is contained in player $i$’s path $\pi_i$, is $c_k$, where $k$ is the total number of players that have chosen edge $e$ in their path. The total cost to player $i$, in the given profile $\pi$, is the sum of the costs of all the edges in its path $\pi_i$ from $s$ to $t$. Each player of course wants to minimize its own total cost.

Compute a pure strategy Nash Equilibrium in this atomic network congestion game.
Figure 1: A 6-state MDP.

Figure 2: Atomic network congestion game