AGTA Tutorial 7

Please attempt the question before your tutorial.

1. Consider the 6-state MDP depicted in Figure 1.

   In this MDP, states s_5 and s_3 are controlled by the player (the controller), whereas the other states are controlled by "nature" (random).

   Suppose the player's objective is to optimize the probability of reaching state s_6 starting at each state s_i, $i = 1, \ldots, 5$.

   Write the Bellman optimality equations for this MDP with this objective.

   Compute the optimal probabilities, p_i^*, of reaching state s_6 starting from state s_i, for $i = 1, \ldots, 5$, and also compute an optimal strategy for the player.

2. Consider the *atomic network congestion game*, with three players, described by the directed graph in Figure 2.

   In this game, every player i (for $i = 1, 2, 3$) needs to choose a directed path from the source s to the target t. Thus, every player i's set of possible actions (i.e., its set of pure strategies) is the set of all possible directed paths from s to t.

   Each edge e is labeled with a sequence of three numbers (c_1, c_2, c_3). Given a profile $\pi = (\pi_1, \pi_2, \pi_3)$ of pure strategies (i.e., s-t-paths) for all three players, the *cost* to player i of each directed edge, e, that is contained in player i's path π_i, is c_k, where k is the total number of players that have chosen edge e in their path. The total cost to player i, in the given profile π, is the sum of the costs of *all* the edges in its path π_i from s to t. Each player of course wants to minimize its own total cost.

   Compute a pure strategy Nash Equilibrium in this atomic network congestion game.

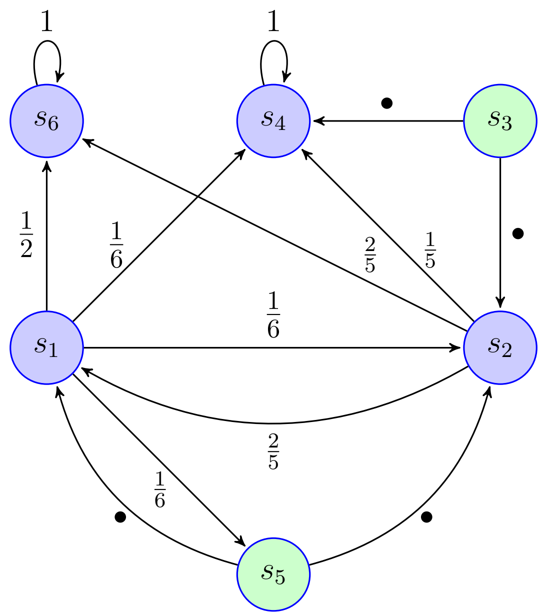

Figure 1: A 6-state MDP.

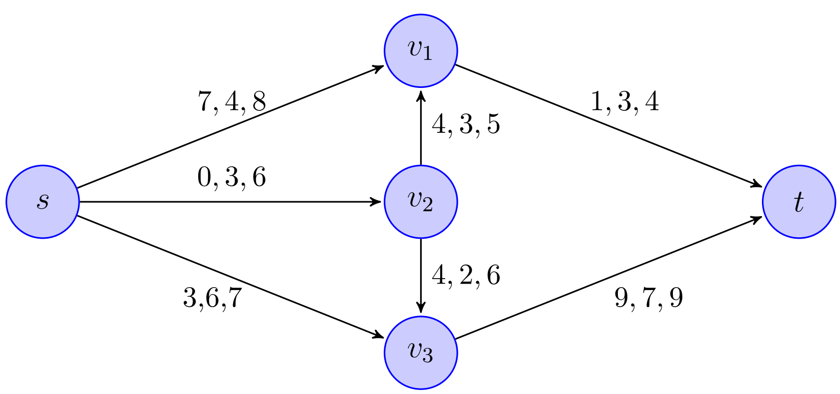

Figure 2: Atomic network congestion game