Algorithmic Game Theory and Applications

Tutorial 4

Attempt to solve the following questions, so that you are able to contribute to the discussion in the tutorial.

1. Consider a 2-player zero-sum game with a payoff matrix A that satisfies $A^T = -A$. Prove that this game has minimax value zero, and that every minmaximizer for Player 1 is also a maxminimizer for Player 2.

2. Consider the following 2-Player game. Each player $i \in \{1, 2\}$ chooses a number $j_i \in \{0, \ldots, 100\}$. The payoff is defined as follows: If $\sum_{i=1}^{2} j_i \leq 100$ then $U_i = j_i$. Otherwise $U_i = 0$.

   Describe all pure Nash Equilibria of this game.
   Find at least one mixed Nash Equilibrium of this game.

3. Find all Nash equilibria of the following 2-Player game in extensive form. First, Player 1 chooses a number $k \in \{0, \ldots, 100\}$. Then Player 2 chooses a number $b \in \{0, \ldots, k\}$. The payoff of Player 1 is $100 - k + 3b$. The payoff of Player 2 is $k - b$.

   Imagine that you are playing the game as Player 1 or Player 2, respectively. What would be your strategy?