AGTA Tutorial Sheet 3

Please attempt these questions before coming to the tutorial.

1. A *symmetric* finite 2-player zero-sum game, is a game that "looks exactly the same" from the point of view of both players. (For example, rock-paper-scissors is a symmetric zero-sum game.)

   More formally, a 2-player zero-sum game is *symmetric* if and only if it is specified by a $(n \times n)$ payoff matrix, $A = (a_{i,j})$, for player 1 (the row player), such that for all $i, j \in \{1, \dots, n\}$, we have $a_{i,j} = -a_{j,i}$, or in order words, such that $A = -A^T$.

   Show that the *minimax value* of any symmetric two player zero-sum game must be zero. (Intuitively, this should be obvious: in a symmetric game neither player can have an advantage, because the game "looks the same" to both players. But I want you to prove it. Hint: prove it by contradiction, assuming the value $v^* \neq 0$.)

2. Consider a 2-player zero-sum matrix game, given by the (2×3) payoff matrix:

$$A = \begin{bmatrix} 2 & 9 & 4 \\ 7 & 0 & 3 \end{bmatrix}$$

   Construct the LP, described in class, for computing the minimax value, and minmaximizer strategy for player 1, for this game. Can you use this LP to compute the minimax value of this zero-sum game, and compute a minmaximizer strategy for player 1?