AGTA Tutorial Sheet 3

Please attempt these questions before coming to the tutorial.

1. A *symmetric* finite 2-player zero-sum game, is a game that "looks exactly the same" from the point of view of both players. (For example, rock-paper-scissors is a symmetric zero-sum game.)

   More formally, a 2-player zero-sum game is *symmetric* if and only if it is specified by a $(n \times n)$ payoff matrix, $A = (a_{i,j})$, for player 1 (the row player), such that for all $i, j \in \{1, \ldots, n\}$, we have $a_{i,j} = -a_{j,i}$, or in order words, such that $A = -A^T$.

   Show that the *minimax value* of any symmetric two player zero-sum game must be zero. (Intuitively, this should obvious: in a symmetric game neither player can have an advantage, because the game "looks the same" to both players. But I want you to prove it. Hint: prove it by contradiction, assuming the value $v^* \neq 0$.)

2. Consider a 2-player zero-sum matrix game, given by the (2×3) payoff matrix:

   $$A = \begin{bmatrix} 2 & 9 & 4 \\ 7 & 0 & 3 \end{bmatrix}$$

   Construct the LP, described in class, for computing the value, and minmaximizer strategy for player 1, for this game. Confirm that the LP looks like this:

   Maximize v
   Subject to:
   $v - 2x_1 - 7x_2 \leq 0$
   $v - 9x_1 - 0x_2 \leq 0$
   $v - 4x_1 - 3x_2 \leq 0$
   $x_1 + x_2 = 1$,
   $x_1 \geq 0$, $x_2 \geq 0$.

   Construct the dual of this LP, according to the "general recipe for LP duals" given in the lecture 7 slides (page 6). Conclude that the dual LP is precisely the LP for computing the value, and maxminimizer strategy for player 2, in the same game.

3. This is the "food for thought" question posed in the lecture slides regarding LP duality:

   recall the "diet problem", described in the first lecture on LPs. It can be put in the form:

   Minimize $c^T x$
   Subject to:
   $Ax \geq b$
   $x_1, \ldots, x_n \geq 0$

   Construct the dual to this LP.

   Then try to interpret the "meaning" of the dual. What do the dual variables "mean", in the context of the diet problem? What is the dual trying to optimize? If the optimizing agent in the primal is "the frugal dieter", then who is the optimizing "counter-agent" in the dual?

   (Hint: try to assign consistent "units of measure" to the primal variables, constants, and coefficients. These, and the constraints of the primal and dual, will then determine what the appropriate "units of measure" for the dual variables are, and that will hopefully guide you toward an interpretation of the dual.)