Algorithmic Game Theory and Applications: Coursework 2

Due into the ITO at 3:00pm on Tuesday, April 10th. (This is a firm deadline.) **Please write your tutorial group number, and your ID number, on the coursework that you hand in.**

1. Recall that a Nash equilibrium in an extensive game is *subgame perfect nash equilibrium* (SPNE) if it is also a Nash equilibrium in every *subgame* of the original game. Formally, a *subgame*, is a game defined by a subtree, T_v of the original game tree, T, such that the subtree T_v, rooted at a node v, has the property that for every decendent u of v in the game tree (including v itself), every node in the same information set as u is also in the subtree T_v.

   (a) [7 points] Give an example of a pure NE which is *not* a SPNE, for a finite extensive form game of perfect information.

   (b) [10 points] Show that every finite extensive game of perfect information where *there are no chance nodes* and where no player has the same payoffs at any two distinct terminal nodes has a *unique* pure-strategy SPNE.

   (c) [8 points] Give an example of a finite extensive form game that contains a pure Nash Equilibrium but does not contain any subgame-perfect pure Nash Equilibrium. Justify your answer.

2. Consider the finite extensive form game of imperfect information depicted in Figure 1.

   (a) [6 points] Translate this game into an equivalent normal form game.

   (b) [7 points] Compute all pure SPNEs in this game.

   (c) [5 points] Are there any mixed Nash equilibria other than the pure SPNEs you have computed? Explain.

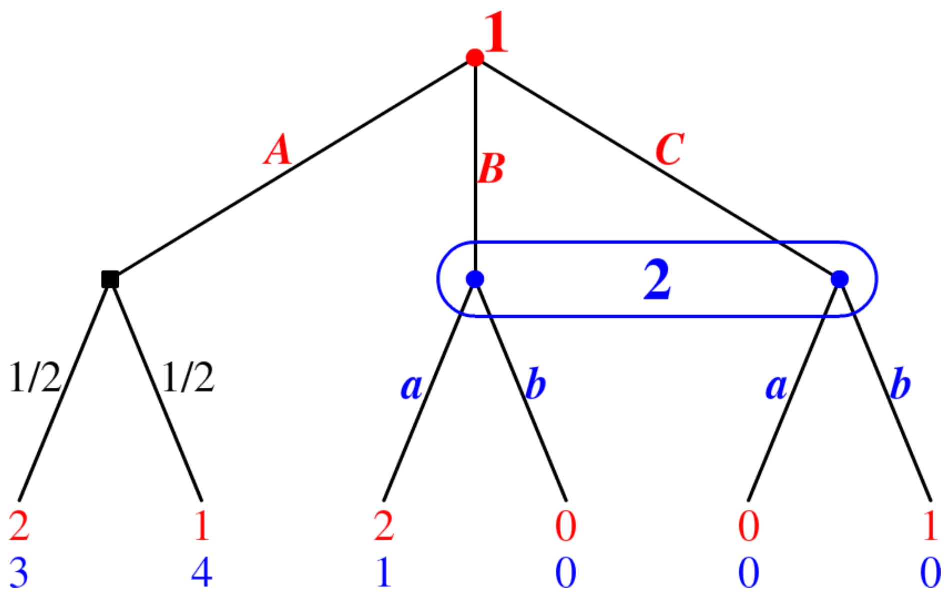

Figure 1: Question 2

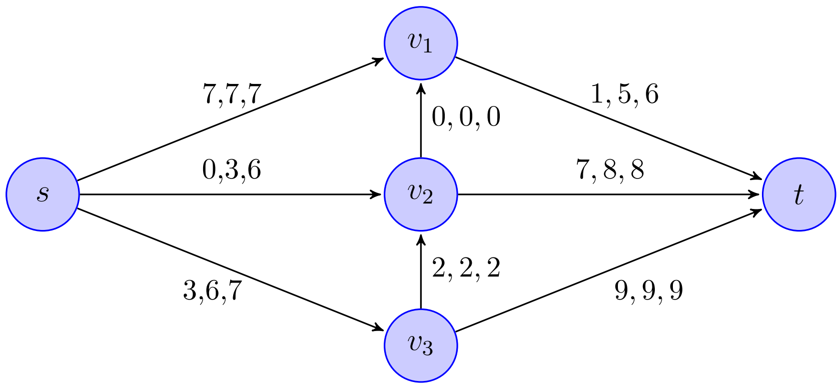

Figure 2: Question 3

(d) [7 points] Which of the equilibria in this game (pure and mixed) are "credible" and "sequentially rational" (specifically, which ones do not involve "non-credible threats"). Explain.

3. Consider the *atomic network congestion game*, with three players, described by the directed graph in Figure 2.

   In this game, every player i (for $i = 1, 2, 3$) needs to choose a directed path from the source s to the target t. Thus, every player i's set of possible actions (i.e., its set of pure strategies) is the set of all possible directed paths from s to t.

   Each edge e is labeled with a sequence of three numbers (c_1, c_2, c_3). Given a profile $\pi = (\pi_1, \pi_2, \pi_3)$ of pure strategies (i.e., s-t-paths) for all three players, the *cost* to player i of each directed edge, e, that is contained in player i's path π_i, is c_k, where k is the total number of players that have chosen edge e in their path. The total cost to player i, in the given profile π, is the sum of the costs of *all* the edges in its path π_i from s to t. Each player of course wants to minimize its own total cost.

   (a) [10 points] Compute a pure strategy NE in this atomic network congestion game. Explain why what you have computed is a pure NE.

   (b) [5 points] Is the pure NE you have computed unique? Explain.

   (c) [10 points] Recall that the *social welfare* of a profile of strategies is the sum of the total cost of all players in the NE. Compute the *price of anarchy* in this atomic network congestion game, where the price of anarchy here is defined as the ratio of the worst social welfare of any NE, divided by the best social welfare of any pure profile.

4. The auction house Christie's of London is auctioning a triptych (a series of three related painting) by the famous artist Fracis Bacon, entitled "Three Studies of Lucian Freud". We will refer to the three paintings in the triptych series as T1, T2, and T3, respectively.

   Suppose that Christie's hires you as an auction designer, and suppose that you decide to use the Vickery-Clarke-Groves mechanism as an auction, in order to determine which bidder should get which part(s) of the triptych, and at what price. Suppose that there are only two bidders. (Nobody else can compete with these two billionaires, so others don't even bother to participate.) The two bidders are: Susanne (S) and Lakshmi (L).

   Given that you are running a VCG-based auction, you ask each of the two bidders to give you their valuation for every subset of the paintings in the triptych, as part of the bidding process. Suppose that the valuation functions v_S and v_L that you receive from the two bidders, S and L, respectively, are as follows (the numbers denote millions of pounds):

	valuation							
bidder i	$v_i(\emptyset)$	$v_i(T_1)$	$v_i(T_2)$	$v_i(T_3)$	$v_i(T_1, T_2)$	$v_i(T_1, T_3)$	$v_i(T_2, T_3)$	$v_i(T_1, T_2, T_3)$
$i := S$	0	20	12	20	31	43	33	64
$i := L$	0	11	30	15	41	30	46	61

   (a) [22 points] What is the outcome of this VCG auction? In other words, which of the two bidders will get which of the painting(s), and what price will they each pay? Justify your answer, and show your calculations.

   (b) [3 points] What if instead of this triptych, Christie's wanted to do a simultaneous auction of 20 Andy Warhol paintings, and they knew that at least 30 viable bidders want to bid for (subsets of) those paintings. Would you suggest using the VCG mechanism for such an auction? What alternative auction would you use, and why? Explain, briefly.