# Algorithmic Game Theory and Applications

Coursework 2

This coursework is due by **4:00pm, on Thursday, 20. November 2014**. (This is a firm deadline. Please hand in your solution, written on paper, by that time to the ITO.) This coursework counts for 15 percent of the overall course mark.

1. (20 points) In arguing the validity of the Lemke-Howson algorithm, we used the trivial fact that in a finite undirected graph where every node has either degree 1 or degree 2, if there exists a node $s$ of degree 1 then there must also exist a different node $t \neq s$ of degree 1 such that $s$ and $t$ are connected (by a path of undirected edges).

   Give a formal proof of this property.

2. (25 points) Consider the following restricted kind of Muller game on a graph. The winner is determined as follows: we are given a set $F \subseteq V$. Every infinite play $\pi$ where $inf(\pi) \cap F \neq \emptyset$ is winning for player 1. All other plays are losing for player 1. Describe an efficient (polynomial time, and as efficient as you can get it) algorithm for determining the winner and computing a memoryless winning strategy in such a game.

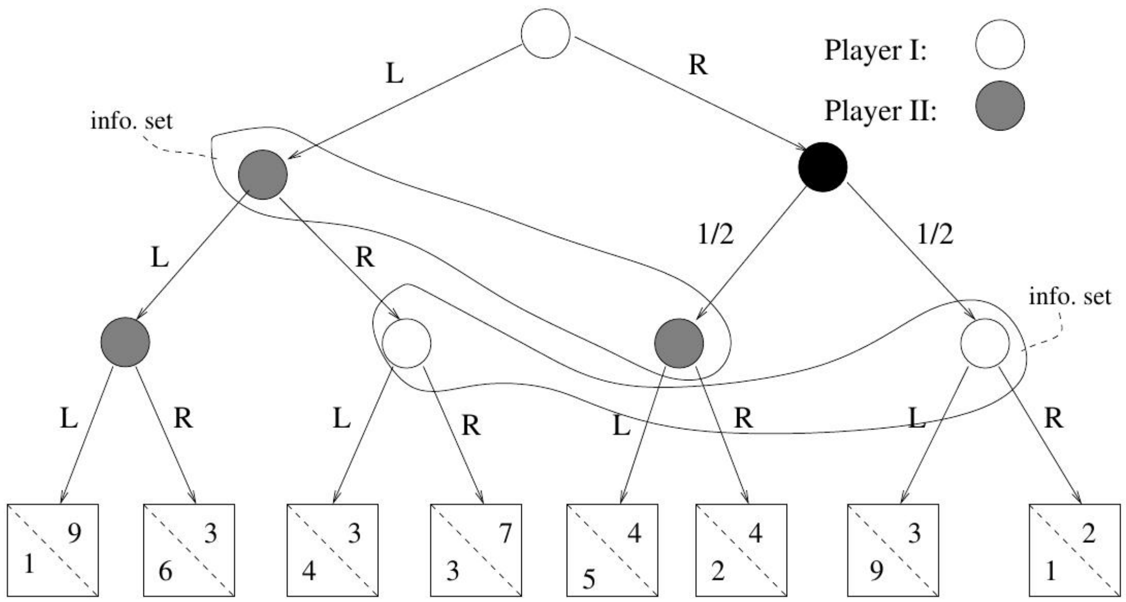
3. (30 points; 10 each) Recall that a Nash equilibrium in an extensive form game is subgame perfect if it is also a Nash equilibrium in every subgame of the original game. Formally, a subgame, is a game defined by a subtree, $T_v$ of the original game tree, $T$, such that the subtree $T_v$, rooted at a node $v$, has the property that for every descendent $u$ of $v$ in the game tree (including $v$ itself), every node in the same information set as $u$ is also in the subtree $T_v$.

   (a) Give an example of a pure NE which is not subgame-perfect, for a finite extensive form game of perfect information.

   (b) Show that every finite extensive form game of perfect information where there are no chance nodes and where no player has the same payoffs at any two distinct terminal nodes has a unique pure strategy subgame perfect Nash equilibrium.

   (c) Give an example of a finite extensive form game that contains a pure Nash Equilibrium but does not contain any subgame-perfect pure Nash Equilibrium. Justify your answer.

4. (25 points) Consider the extensive form game provided in the Figure below. Find a Nash Equilibrium for this game and the expected payoffs to each player under this NE. (In leaves, payoffs in the lower left are for player 1.)

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