Algorithmic Game Theory and Applications: Homework 2

November 9, 2009

This homework is due in to the ITO by 3:00pm, Friday, November 27th.

1. Consider the extensive form game provided in the Figure 1. Find a Nash Equilibrium for this game and the expected payoffs to each player under this NE. (In leaves, payoffs in the lower left are for player 1.)
2. Recall that a Nash equilibrium in an extensive game is *subgame perfect* if it is also an equilibrium in every *subgame* of the original game.

   Formally, a *subgame*, is a game defined by a subtree, T_v of the original game tree, T, such that the subtree T_v is rooted at a node, v, which is in an information set of size 1 (i.e., the information set only contains v), and furthermore, such that all of v's decendents, u, have the property that that ever other node in the same information set as u is also a decendent of v.

   (a) Give an example of an NE for a finite extensive form game which is not subgame perfect.

   (b) Show that every finite game of perfect information where *there are no chance nodes* and where no player has the same payoffs at any two terminal nodes has a *unique* pure strategy subgame perfect Nash equilibrium.

   (c) (Harder) Does every finite extensive form game (not necessarily of perfect information), contains a subgame perfect (mixed) Nash equilibrium? Justify your answer.

3. Consider the two person extensive game depicted in Figure 3. Find a pure *subgame perfect* Nash Equilibrium for this game and the expected payoffs to each player under this NE.
4. In arguing the validity of the Lemke-Howson algorithm, we used the trivial fact that in a finite undirected graph where every node has

either degree 1 or degree 2, if there exists a node s of degree 1 then there must also exist a different node $t \neq s$ of degree 1 such that s and t are connected (by a path of undirected edges). Prove this trivial fact.

5. Consider the following restricted kind of Muller game on a graph. The winner is determined as follows: we are given a set $F \subseteq V$. Every infinite play π where $\inf(\pi) \cap F \neq \emptyset$ is winning for player 1. All other plays are losing for player 1. (Such games are call Büchi games.) Describe an efficient (polynomial time, and as efficient as you can get it) algorithm for determining the winner and computing a memoryless winning strategy in such a game.

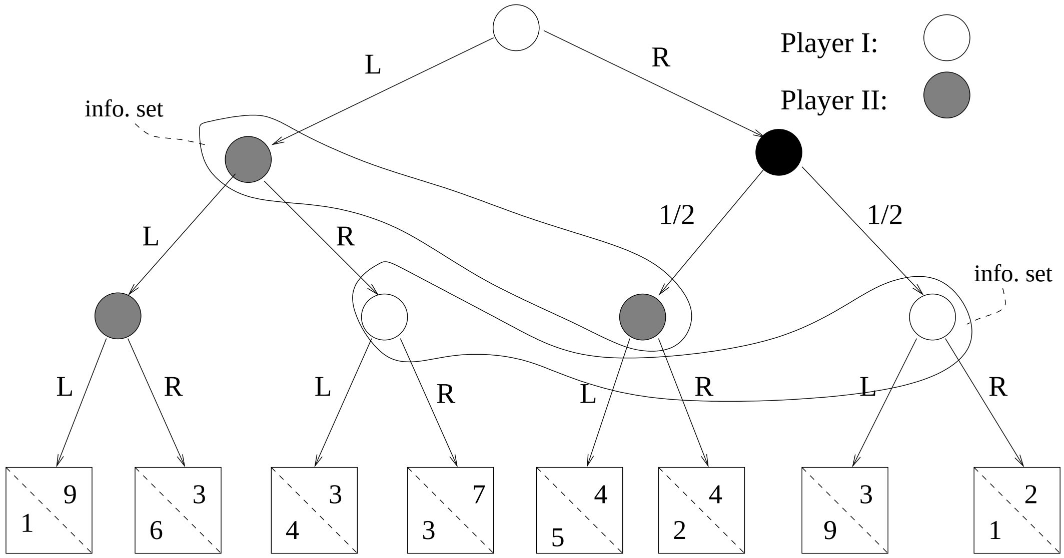

Figure 1: problem 1

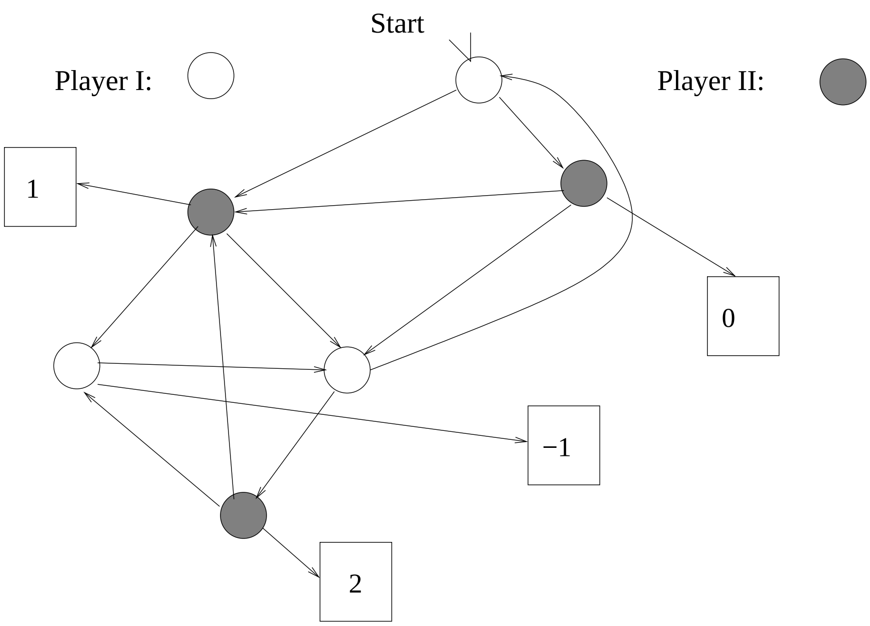

Figure 2: problem 2

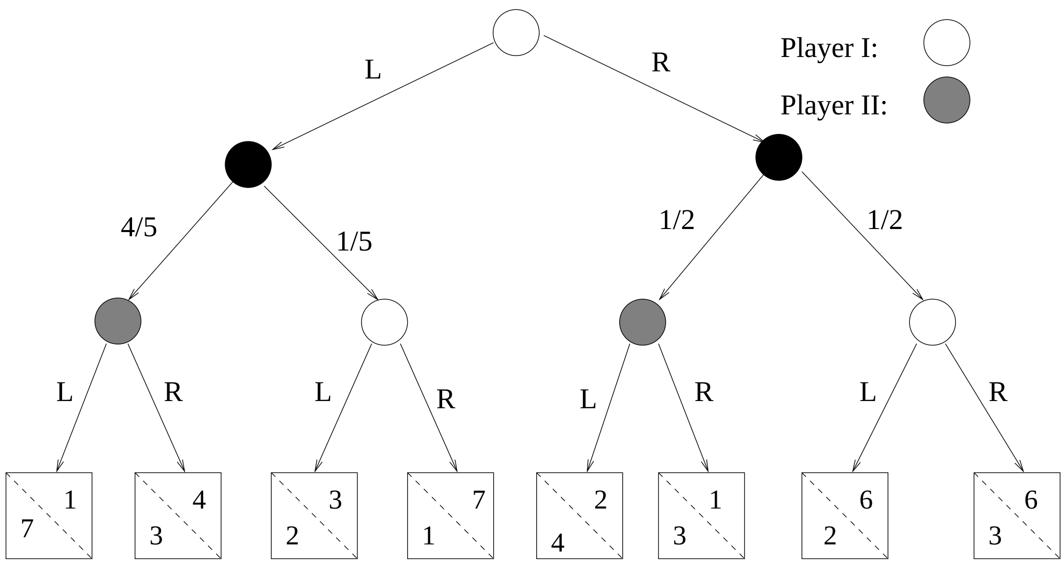

Figure 3: problem 3