Algorithmic Game Theory and Applications

Coursework 1

This coursework is due by **4:00pm, on Thursday, October 30**. (This is a firm deadline. Please hand in your solution, written on paper, by that time to the ITO.) Each question counts for 25 points, for a total of 100 points. This coursework counts for 15 percent of the overall course mark.

1. Consider the following 2-player strategic game, G:

$$\begin{bmatrix} (6,5) & (4,8) & (6,4) & (9,2) \\ (4,6) & (7,4) & (7,5) & (4,4) \\ (4,7) & (4,4) & (9,5) & (2,6) \\ (5,9) & (4,10) & (4,9) & (8,9) \end{bmatrix}$$

   This is a "bimatrix", to be read as follows: Player 1 is the row player, and Player 2 is the column player. If the content of the bimatrix at row i and column j is the pair (a,b), then $u_1(i,j) = a$ and $u_2(i,j) = b$. Find (i.e., compute) all Nash equilibria for this game G. For any profile x that you claim is an NE of G, prove that x is indeed an NE of G. Prove there are no other (pure or mixed) NEs, other than the profiles you claim are NEs.

2. In Brouwer's fixed point theorem, one makes several assumptions about the function

$$f : D \to D$$

   and the space $D \subseteq \mathbb{R}^m$.

   (a) f is continuous.
   (b) D is bounded.
   (c) D is closed.
   (d) D is convex.

   Prove that each of these four conditions is necessary for the theorem to hold (i.e., for f to have a fixed point in D). For each condition give a counterexample that fails this condition and the fixpoint property, but still satisfies all the other conditions.

3. Consider the 2-player zero-sum game given by the following payoff matrix for player 1 (the row player):

$$\begin{bmatrix} 4 & 2 & 1 & 9 & 5 \\ 8 & 3 & 5 & 6 & 2 \\ 2 & 4 & 6 & 1 & 7 \\ 4 & 9 & 3 & 4 & 6 \\ 2 & 5 & 6 & 4 & 1 \end{bmatrix}$$

   Compute the minimax value of this game, and also find a minimax profile, i.e., "optimal" (minmaximizer and maxminimizer) strategies for players 1 and 2, respectively. (You can use computer algebra tools. See http://www.sagemath.org/ for an open-source computer algebra package. The LP-solver "ppl" works over the rationals, i.e., without rounding.) Also specify the LPs you used to solve the game.

4. (a) (13 points.) Note that a Linear Programming problem need not have a unique optimal solution. There may be many optimal solutions (in fact there may be infinitely many optimal solutions). Consider the following "revised LP problem": the LP is given by constraint set $C(x_1, \ldots, x_n)$, together with an ordered listing of linear objective functions together with optimization criteria for each as follows: $((f_1, Opt_1), \ldots, (f_r, Opt_r))$. Here each f_j is a linear objective and each Opt_j is either Maximize or Minimize. The interpretation of this "revised LP problem" is as follows: you wish to find a solution vector $v \in K(C)$, such that: v optimizes f_1 according to criterion Opt_1, and, among all vectors that do so, v optimizes f_2 according to criterion Opt_2, and among all vectors that do so, etc. Give an algorithm for solving this revised LP problem.

   (b) (12 points.) Find the dual to the following LP:

   Maximize $2x_1 + 3x_2 + 5x_3$
   Subject to:
   $2x_1 + x_2 \leq 5$
   $3x_1 + 4x_3 \leq 6$
   $4x_2 + 5x_3 \leq 8$
   $x_1, x_2, x_3 \geq 0$

   Then, solve the LP and its dual, i.e., find the optimal value and the instantiations (for the LP and for its dual) that achieve it.
   (You can use computer algebra tools, e.g., http://www.sagemath.org/).

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