1. Consider the following typesetting problem. The input is a sequence of \( n \) words containing \( l_1, l_2, \ldots, l_n \) characters, respectively. Each line can hold at most \( P \) characters, the text is left-aligned, and words cannot be split between lines. If a line contains words from \( i \) to \( j \) (inclusive) then the number of spaces at the end of the line is \( s = P - \sum_{k=i}^{j} l_k - (j - i) \) (because the words are separated by white spaces). We aim to typeset the text so as to avoid large white spaces at the end of lines, i.e., we would like to minimise the sum over all lines of the square of the number of white spaces at the end of the line.

   (a) Give an efficient algorithm for this problem. (25 points)
   
   (b) Formally prove the running time of the algorithm (matching upper and lower bounds). (15 points)

2. (30 points) Let \( x_1, \ldots, x_n \) be (not necessarily distinct) integers in the range \( \{1, \ldots, n\} \). Let \( (a_1, b_1), \ldots, (a_k, b_k) \) be pairs of integers. Furthermore, for any \( 1 \leq i \leq k \) let \( Z_i \) be the number of indices \( j \) such that \( a_i \leq x_j \leq b_i \). Devise an algorithm that, given \( x_1, \ldots, x_n \) and \( (a_1, b_1), \ldots, (a_k, b_k) \), computes the numbers \( Z_1, \ldots, Z_k \) in time \( O(n + k) \).

3. (30 points) Prove or disprove the following statement: There exists a comparison-based sorting algorithm whose running time is linear for at least a fraction of \( 1/2^n \) of the \( n! \) possible input instances of length \( n \).