Algorithms and Data Structures 2020/21

Coursework 1

This coursework is due by 4:00pm, on Friday, 23. Oct. 2020 (upload on the LEARN page (www.learn.ed.ac.uk) of ADS 2020/21). This is a firm deadline. This coursework 1 is formative.

1. Recurrence sequences.

(a) Solve the recurrence $T(n) = 3T(\sqrt{n}) + \log n$ by making a change of variables. Your solution should be asymptotically tight. Do not worry about whether values are integral. (10 points)

(b) Prove a good (as good as you can manage) asymptotic upper bound on the recurrence $T(n) = T(n-1) + T(n/2) + n$. Use the substitution method to verify your answer. (20 points)

(c) Suppose you want to develop a matrix-multiplication algorithm that is asymptotically faster than Strassen’s algorithm. Your algorithm will use the divide-and-conquer method, dividing each matrix into pieces of size $n/4 \times n/4$, and the divide and combine steps together will take $\Theta(n^2)$ time. How many subproblems would your algorithm need to create (at most) in order to be asymptotically faster than Strassen’s algorithm? (10 points)

2. DFT.

(a) Compute the DFT of the vector $(0, 1, 2, 5)$. (10 points)

(b) One way to evaluate a polynomial $A(x)$ of degree-bound $n$ at a given point $x_0$ is to divide $A(x)$ by the polynomial $x - x_0$, obtaining a quotient polynomial $q(x)$ of degree-bound $n - 1$ and a remainder $r$, such that $A(x) = q(x)(x - x_0) + r$. Clearly, $A(x_0) = r$. Show how to compute the remainder $r$ and the coefficients of $q(x)$ in time $\Theta(n)$ from $x_0$ and the coefficients of $A$. (10 points)

(c) Given a list of values $z_0, z_1, \ldots, z_{n-1}$ (possibly with repetitions), show how to find the coefficients of a polynomial $P(x)$ of degree-bound $n + 1$ that has zeros only at $z_0, z_1, \ldots, z_{n-1}$ (possibly with repetitions). Your procedure should run in time $O(n \log^2 n)$. (20 points)

(d) Consider two sets $A$ and $B$, each containing $n$ integers in the range from 0 to 10n. We wish to compute the Cartesian sum of $A$ and $B$, defined by

$$C := \{ x + y \mid x \in A \text{ and } y \in B \}$$

Note that the integers in $C$ are in the range from 0 to 20n. We want to find the elements of $C$ and the number of times each element of $C$ is realized as a sum of elements in $A$ and $B$. Show how to solve the problem in $O(n \log n)$ time. (20 points)

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