Algorithms and Data Structues 2020/21

Coursework 1

This coursework is due by **4:00pm, on Friday, 23. Oct. 2020** (upload on the LEARN page (www.learn.ed.ac.uk) of ADS 2020/21). This is a firm deadline. This coursework 1 is **formative**.

1. Recurrence sequences.

   (a) Solve the recurrence $T(n) = 3T(\sqrt{n}) + \lg n$ by making a change of variables. Your solution should be asymptotically tight. Do not worry about whether values are integral. **(10 points)**

   (b) Prove a **good** (as good as you can manage) asymptotic upper bound on the recurrence $T(n) = T(n-1) + T(n/2) + n$. Use the substitution method to verify your answer.
   . **(20 points)**

   (c) Suppose you want to develop a matrix-multiplication algorithm that is asymptotically faster than Strassen's algorithm. Your algorithm will use the divide-and-conquer method, dividing each matrix into pieces of size $n/4 \times n/4$, and the divide and combine steps together will take $\Theta(n^2)$ time. How many subproblems would your algorithm need to create (at most) in order to be asymptotically faster than Strassen's algorithm? **(10 points)**

2. DFT.

   (a) Compute the DFT of the vector $(0, 1, 2, 5)$. **(10 points)**

   (b) One way to evaluate a polynomial $A(x)$ of degree-bound n at a given point x_0 is to divide $A(x)$ by the polynomial $x - x_0$, obtaining a quotient polynomial $q(x)$ of degree-bound $n - 1$ and a remainder r, such that $A(x) = q(x)(x - x_0) + r$. Clearly, $A(x_0) = r$. Show how to compute the remainder r and the coefficients of $q(x)$ in time $\Theta(n)$ from x_0 and the coefficients of A. **(10 points)**

   (c) Given a list of values $z_0, z_1, \ldots, z_{n-1}$ (possibly with repetitions), show how to find the coefficients of a polynomial $P(x)$ of degree-bound $n + 1$ that has zeros only at $z_0, z_1, \ldots, z_{n-1}$ (possibly with repetitions). Your procedure should run in time $O(n \lg^2 n)$. **(20 points)**

   (d) Consider two sets A and B, each containing n integers in the range from 0 to $10n$. We wish to compute the Cartesian sum of A and B, defined by

   $$C := \{x + y \mid x \in A \text{ and } y \in B\}$$

   Note that the integers in C are in the range from 0 to $20n$. We want to find the elements of C and the number of times each element of C is realized as a sum of elements in A and B. Show how to solve the problem in $O(n \lg n)$ time. **(20 points)**

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