Below are a list of suggested exercises. You should also see the tutorial as a resource to get answers to questions you have, don’t feel compelled to stick to the sheet.

1. Draw an example of a weighted graph which has 2 MSTs.

2. Let $G, W$ be a weighted graph in which all edge weights are distinct. Prove that the MST of $G, W$ is unique.

3. In line 3 of Prim’s algorithm, there may be more than one fringe edge of minimum weight. Suppose we add all these minimum edges in one step. Does the algorithm still compute a MST?

4. Consider an arbitrary edge $(u, v)$ in a graph $G$. There may or may not be an MST of $G$ which contains $(u, v)$, depending on the edge weights given by $W$.

Give a $O(|V| + |E|)$ time algorithm to determine, for a given weighted graph $(G, W)$, and a given edge $(u, v)$ of $G$, whether $(u, v)$ belongs to some MST of $G$. Justify your algorithm!