Algorithms and Data Structures 2020/21
Week 3 tutorial sheet

Below are a list of *suggested* exercises, some from the back of either lecture 1, 2 or 3). Ask your tutor to cover the questions most important to you. You should see the tutorial as a resource to get answers to any questions, so don't feel compelled to stick to the sheet.

1. Work out (don't bother proving) for each pair of expressions below, whether A is $O(B)$, $\Omega(B)$, $\Theta(B)$ (it could be none of these). Assume $k \geq 1, \epsilon > 0, c > 1$ are all constants.

A	B	O	Ω	Θ
$\lg^k n$	n^ϵ			
n^k	c^n			
$\sqrt{n}$	$n^{\sin n}$			
2^n	$2^{n/2}$			
$n^{\ln m}$	$m^{\ln n}$			
$\lg(n!)$	$\lg(n^n)$			

2. Define

$$p(n) = \sum_{i=0}^{d} a_i n^i,$$

where $a_d > 0$. So $p(n)$ is a degree-d polynomial in n. Let k be some constant.

Prove the following *by first principles*, from the definitions of $O(\cdot)$, $\Omega(\cdot)$ and $\Theta(\cdot)$.

(a) If $k \geq d$, then $p(n) = O(n^k)$.

(b) If $k \leq d$, then $p(n) = \Omega(n^k)$.

(c) If $k = d$, then $p(n) = \Theta(n^k)$.

3. Analyse the asymptotic worst-case running time of the three POWER-REM algorithms from Lecture 1.