

**Algorithms and Data Structures 2015/16**  
**Week 8 tutorial sheet (Tues 8th - Friday 12th March)**

Below are a list of *suggested* exercises. You should also see the tutorial as a resource to get answers to questions you have, don't feel compelled to stick to the sheet.

1. Draw an example of a weighted graph which has 2 MSTs.
2. Let  $G, \mathcal{W}$  be a weighted graph in which all edge weights are distinct.  
Prove that the MST of  $G, \mathcal{W}$  is unique.
3. In line 3 of Prim's algorithm, there may be more than one fringe edge of minimum weight. Suppose we add all these minimum edges in one step. Does the algorithm still compute a MST?
4. Consider an *arbitrary* edge  $(u, v)$  in a graph  $G$ . There may or may not be an MST of  $G$  which contains  $(u, v)$ , depending on the edge weights given by  $\mathcal{W}$ .

Give a  $O(|V| + |E|)$  time algorithm to determine, for a given weighted graph  $(G, \mathcal{W})$ , and a given edge  $(u, v)$  of  $G$ , *whether*  $(u, v)$  belongs to some MST of  $G$ .

*Justify* your algorithm!

Mary Cryan