

**Algorithms and Data Structures 2011/12**  
**Week 10 tutorial sheet (Tues 22nd - Fri 25th Nov)**

Below are a list of *suggested* exercises. You should also see the tutorial as a resource to get answers to questions you have, don't feel compelled to stick to the sheet.

1. Show how to determine in  $O(n^2 \lg n)$  time whether any three points in a set of  $n$  points are co-linear.

*This is Ex. 33.1-4 of [CLRS].*

2. Given a point  $p_0 = (x_0, y_0)$ , the *right horizontal ray* from  $p_0$  is the set of points  $\{p = (x, y_0) : x \geq x_0\}$ , that is, it is the set of points due right of  $p_0$ . Show how to determine whether a right horizontal ray from a given  $p_0$  intersects a line segment  $\overline{p_1 p_2}$  in  $O(1)$  time, by reducing the problem to that of two line segments intersecting.

*This is Ex. 33.1-6 of [CLRS]. Ex. 35.1-5 of [CLR].*

3. Show that there may be  $\Theta(n^2)$  intersections in a set of  $n$  line segments.

*This is Ex. 33.2-1 of [CLRS]. Ex. 35.2-1 of [CLR].*

4. In the *online convex hull problem*, we are given the set  $Q$  of  $n$  points one point at a time. After receiving each point, we are to compute the convex hull of the points seen so far. Obviously, we could run Graham's scan once for each point, with a total running time of  $O(n^2 \lg n)$ . Show how to improve this slightly, by showing we can solve the online convex hull problem in  $O(n^2)$ .

*This is Ex. 33.3-5 of [CLRS]. Ex. 35.3-5 of [CLR].*

5. Prove that the problem of finding the Convex Hull of  $n$  points (in a "comparison-based" fashion) has a lower bound of  $\Omega(n \lg n)$ . For this, think about using a *reduction* from sorting to Convex Hull (that is, think about how to use a Convex Hull algorithm to sort a list of numbers).

Mary Cryan