proving \( C(n) \), the worst-case # of comparisons of QuickSort, is \( O(n^2) \), proposed as CLASS discussion, ended up dominating the lecture!

Answer to "How" is proof by induction

**CLAIM:** \( C(n) \leq \frac{n(n+1)}{2} \) \( \forall n \in \mathbb{N} \).

**proof by induction**

**base cases** \( C(1) = 1 \) \( \left( \leq \frac{1 \cdot 2}{2} \right) \)

\( C(2) = 2 \) \( \left( \leq \frac{2 \cdot 3}{2} \right) \)

**induction step**

Suppose true for all \( m < n \). \( \text{(I.H.)} \)

Consider

\[
C(n) = \max \left\{ C(k-1) + C(n-k) \right\} + n
\]

\[
\leq \max_{1 \leq k \leq n} \left\{ \frac{(k-1)k}{2} + \frac{(n-k)(n-k+1)}{2} \right\} + n
\]

\[
= \max_{1 \leq k \leq n} \left\{ \frac{2k^2 - 2k + n^2 + n - 2kn}{2} \right\} + n
\]

\[
= \max_{1 \leq k \leq n} \left\{ \frac{2k^2 - 2k + n^2 + 3n - 2kn}{2} \right\}
\]

\[
= \max_{1 \leq k \leq n} \left\{ \frac{n^2 + n}{2} + k^2 - k - kn + n \right\}
\]

\[
= \max_{1 \leq k \leq n} \left\{ \frac{n(n+1)}{2} + n - k\left[(n+1)-k\right] \right\}
\]

but \( k(n+1-k) > n \) for all \( 1 \leq k \leq n \)

\[
\rightarrow \quad n - k(n+1-k) < 0 \quad \text{for all} \quad 1 \leq k \leq n
\]

\[
\rightarrow \quad C(n) \leq \max_{1 \leq k \leq n} \left\{ \frac{n(n+1)}{2} \right\} = \frac{n(n+1)}{2} = O(n^2)
\]