Using Linked Lists with "Weighted Union"

\( \hat{m} \) operations, \( \hat{n} \) of which are "Make-Set".

- We know that at most \((\hat{n} - 1)\) of the \( \hat{m} \) operations can be Union operations.
  - Because after doing \((\hat{n} - 1)\) Union operations on disjoint sets over \( \hat{n} \) elements, we are guaranteed that all elements belong to a single set.

- "Make-Set" and "Find-Set" take \( \Theta(1) \) time per operation, so the time for our \( \hat{m} \) operations is
  \[
  \Theta(\hat{m}) + \text{"Time for all the Union operations"} = \Theta(\hat{m}) + \Theta(\hat{n} \log \hat{n})
  \]

"Time for all the Union operations"

\[
\sum_{\text{all Union operations}} \Theta(\text{length of shorter list in union})
\]

\[
= \sum_{\text{all Union operations}} \sum_{\text{all } z \text{ in shorter list of Union}} \Theta(1)
\]

\[
= \sum_{\text{all elements } z} \sum_{\text{all Unions where } z \text{ is in the shorter list}} \Theta(1) = \Theta(\hat{n} \log \hat{n})
\]

WHERE THE FINAL STEP FOLLOWS BECAUSE ANY \( z \) CAN ONLY BE IN THE SHORTER LIST OF A UNION AT MOST \( (\log \hat{n}) \) TIMES, AND BECAUSE THERE ARE ONLY \( \hat{n} \) "z"s.