Agent-Based Systems Tutorial 8

Version with suggested solutions

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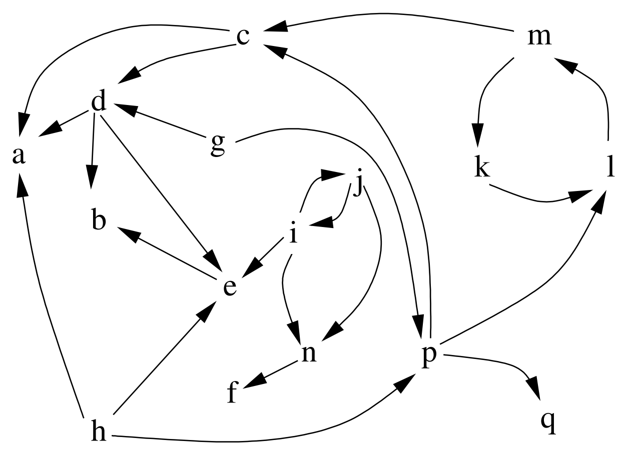
Suggestions for solutions and hints are printed in italics below each question

Q1 Consider the abstract argumentation system depicted in the following graph:



1. Construct the grounded extension.
2. Construct the preferred extension(s).
3. Which arguments can be credulously justified?
4. Which arguments can be sceptically justified?

Solution suggestions:

1. The grounded extension of an argumentation system is the least fixed point of the characteristic function $\mathcal{F}$. The *characteristic function* of an argumentation system $\mathcal{A} = \langle X, \rightarrow \rangle$, is the function $\mathcal{F} : 2^X \rightarrow 2^X$, which is defined as follows:

$$\mathcal{F}(S) = \{a \mid a \textit{ is acceptable } \text{w.r.t. } S\}.$$

The grounded extension can be built incrementally:

(a) Arguments that are not attacked are "in"
(b) Delete from the graph every argument that is attacked by an argument that is in the grounded extension and go to Step 1

- Iterate until there are no more changes to the argument graph

The grounded extension is the set $\{b, g, h, q\}$.

- The arguments g and h have no attackers, so they are "in".
- g attack d and p, and h attacks a, e and p.
- h attacks p, which is the only argument attacking q, so q is "in".
- g attacks d and h attacks e, which are the only argument attacking b is "in".
- There are no more arguments attacked by arguments in the grounded extension.

2. A set of arguments S is conflict-free if if there are no arguments a, b in S such that a attacks b. An argument a is acceptable with respect to a set S of arguments iff for each argument a': if a' attacks a then a' is attacked by some argument in S. A conflict-free set of arguments S is admissible iff each argument in S is acceptable w.r.t. S. Preferred extensions are maximal (w.r.t. set inclusion) admissible sets.

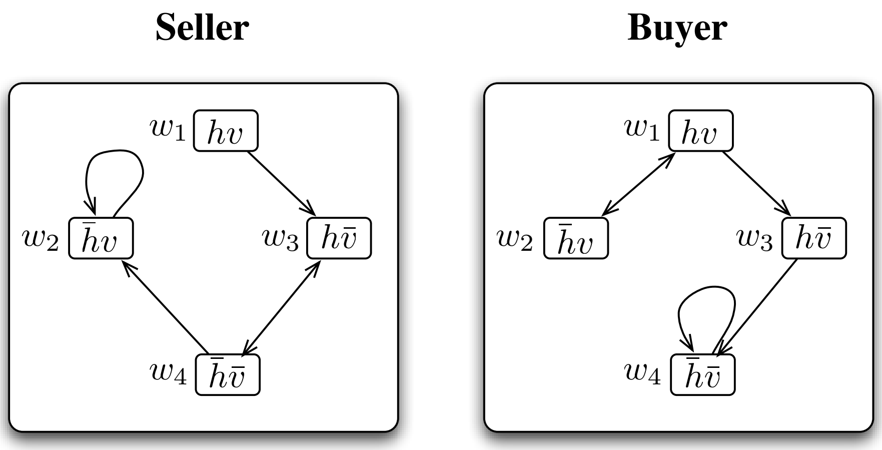
   There are two preferred extensions $\{b, f, g, h, i, q\}$ and $\{b, f, g, h, j, q\}$.

   - The preferred extensions are admissible sets in which we can add no more arguments and they can still be admissible. Therefore, other admissible sets, as for instance $\emptyset$ or $\{g, h\}$, are not preferred extensions.
   - Note that the grounded extension is part of both preferred extensions
   - The different arguments in the two preferred extensions are arguments i and j. These arguments are mutually attacking each other. Each extensions selects one of these.
   - f is only attacked by n, which is attacked both by i and j.

3. An argument is credulously accepted if it is a member of at least one preferred extension. The arguments b, f, g, h, i, j, q are credulously accepted.

4. An argument is sceptically accepted if it is a member of every preferred extension. The arguments b, f, g, h, q are sceptically accepted. Note that f is sceptically accepted because, regardless of whether we accept i or j, f is can be defended against its attackers. A sceptical agent will not chose between i or j, but will accept f, since one of them should be acceptable (as the only attacks that are relevant w.r.t. i and j are their mutual attacks), and f is defended regardless of which one is selected.

Q2 You are given the following two accessibility relations for the beliefs of the Buyer and Seller agents in a domain with two arguments h and v, using a modal logic of belief. The diagrams describe models M_B and M_S with valuation functions π_S and π_B, such that $\Box\varphi$ is interpreted as $(\mathsf{Bel}\ i\ \varphi)$ for each of the two agents $i \in \{B, S\}$):



(a) Which of the T, D, 4 and 5 axioms are satisfied by the Bel modality for each of the two agents?

(b) Which of the following statements is true?

1. $\langle M_S, w_1\rangle \models \neg(h \Rightarrow v)$
2. $\langle M_S, w_1\rangle \models (\text{Bel } S\ \neg v)$
3. $\langle M_B, w_2\rangle \models \neg(\text{Bel } B\ h)$
4. $\langle M_S, w_4\rangle \models (\text{Bel } S\ \neg h \vee \neg v)$
5. $\langle M_B, w_2\rangle \models (\text{Bel } S\ (\text{Bel } S\ \neg h \vee \neg v))$
6. $\langle M_S\rangle \models \neg(\text{Bel } S\ h \Rightarrow v)$
7. $\models (\text{Bel } S\ h \vee \neg h) \wedge (\text{Bel } B\ v \vee \neg v)$
8. $\langle M_B, M_S, w_3\rangle \models E(\neg h \vee \neg v)$
9. $\langle M_B, M_S, w_1\rangle \models D(h \wedge \neg v)$

Justify your answers.

(c) Complete the diagrams so that they satisfy the KD45 axiom system by drawing additional edges between possible worlds.

(d) What further edges are needed if you switch from KD45 to S5?

Solution suggestions:
Part (a): Only the D axiom (seriality) is satisfied, since every world has at least one other world that is accessible from it. The relations are neither reflexive (T), transitive (4), or Euclidean (5) for either of the two agents.
Part (b):

1. *False, because $v \in \pi_S(w_1)$, and if v is true, so is $h \Rightarrow v$*
2. *True, because $\neg v$ is true in the only accessible world w_3 (w_1 is not accessible from w_1 in this example!)*
3. *False, because world w_1 is accessible from w_2 that satisfies h (don't be lead into assuming that w_2 is also accessible and that this would be a world in which h is not the case causing disbelief)*
4. *True, both accessible worlds w_2 and w_3 make either of the two propositions false*

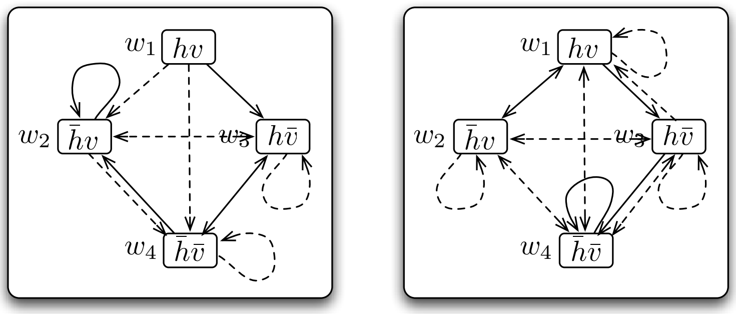
5. *True. The only one-step accessible world is w_1, and every world that is accessible from that world is makes $\neg v$ or $\neg h$ true*
6. *True: there is a world w_1 from which w_3 is accessible and that world does not satisfy $h \Rightarrow v$; therefore the formula is not true in all worlds, and the statement is false.*
7. *True regardless of the models used, since the formulae are tautologies of propositional logic*
8. *True, since $\langle M_B, w_3 \rangle \models (\textit{Bel}\, B\, \neg h \vee \neg v)$ and $\langle M_S, w_3 \rangle \models (\textit{Bel}\, S\, \neg h \vee \neg)$*
9. *True: the only shared link is that from w_1 to w_3 and it satisfies the proposition (thus someone who has distributed knowledge can do away with B's doubts).*

Part (c): To complete the diagrams according to KD45, we need to:

- *Draw an edge from w to w'' whenever there exist links (w, w') and (w', w'') (transitivity/axiom 4); this involves drawing loops (w, w) when there exists a path from w to w.*
- *Draw an edge from w' to w'' whenever there exist links (w, w') and (w, w'') (Euclidean relation/axiom 5); since this can be read in two different ways, we also have to insert a link (w'', w').*
- *Apply these rules recursively, i.e. insert further edges that result from these rules using the new edges already inserted in previous steps.*

We obtain the following diagram (new edges are shown using dashed arrows):

Seller **Buyer**



Part (d): To satisfy the additional axiom T we need to draw one more loop from w_1 to w_1, as it is already satisfied otherwise:

Seller **Buyer**