Agent-Based Systems Tutorial 7

Version with suggested solutions

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Suggestions for solutions and hints are printed in italics below each question

Q1 Prove the following statement: "Bidding one's own valuation in a Vickrey auction is the dominant strategy for a rational agent."
You can assume we are only considering private value auctions among purely self-interested, rational and risk-neutral agents.

Solution suggestions: *We consider agent A with (true) valuation v of the good that is being auctioned. Suppose A bids $w < v$, then some other agent might win the auction with a bid x such that $w < x < v$ and will have to pay a price of at most x, i.e. A lost the auction although he would not have paid more than v which is irrational. Now, suppose A bids $w > v$, then w might be the highest bid and the second highest bid might be x where $w > x > v$ so that x becomes the buying price for A, so that he will have to pay more than v and, hence, lose money.*
Since both bidding below and above A's true valuation v are not rational v remains as the only rational choice.

Q2 Discuss which of the English, Vickrey, first-price sealed bid, or Dutch auction protocols guards better against bidder collusion.

Solution suggestions: *Basically, none of the protocols is fully collusion proof: bidders can always reach an agreement to keep prices artificially low. The English and Vickrey auctions are actually even worse than the Dutch or first-priced sealed bid protocols, since some collusion agreements are self-enforcing. This can be seen from the following example taken from Sandholm (1999): Let bidder A have valuation 20 for the good to be auctions while all others have valuation 18. The bidders can agree to bid 5 while A bids 6. In the English auction, if they don't keep their promise and bid more than 5 A can observe this and continue bidding until reaching 20. In the Vickry auction, A can safely bid 20 since he will get the item at the price of 5 anyway, and no bid between 5 and 18 would win the auction, so the remaining agents have no incentive to bid in that range.*

Q3 Consider the following distribution of utilities for two agents 1 and 2 in a task-oriented negotiation domain:

Deal	$cost_1$	$cost_2$
Θ	5	5
d_1	4	1
d_2	0	4
d_3	2	2
d_4	2	3

Trace the way negotiation would proceed using the monotonic concession protocol in this example if agents used the Zeuthen strategy (you may assume that in the first round, agent 2 proposes deal d_1 and agent 1 proposes deal d_2).

Solution suggestions: *We first translate the above matrix to a utility matrix by taking* $u_i(\delta) = c_i(\Theta) - cost_i(\delta)$ *which yields the following matrix:*

Deal	u_1	u_2
Θ	*0*	*0*
d_1	*1*	*4*
d_2	*5*	*1*
d_3	*3*	*3*
d_4	*3*	*2*

In the first round, agent 2 proposes deal d_1 and agent 1 proposes deal d_2. $u_1(d_1) = 1$ and $u_1(d_2) = 5$ so agent 1 won't accept d_1 and $u_2(d_2) = 1$ and $u_2(d_1) = 4$ so agent 2 won't accept the d_2 either. The risk values that result from this are $risk_1^0 = (5 - 1)/5 = 0.8$ and $risk_2^0 = (4 - 1)/4 = 0.75$. Therefore, agent 2 should concede. There are two ways in which he may concede:

- Proposing d_3: this is admissible since $u_1(d_3) = 3 > 1 = u_1(d_1)$, but agent 1 will still not accept (and continues proposing d_2). The new risk values are $risk_1^1 = (5 - 3)/5 = 0.4$ and $risk_2^1 = (3 - 1)/3 = 0.\bar{6}$. In this case agent 1 has to concede in the next round so that this changes the balance of risk.
- Proposing d_4: this is also admissible, and agent 1 will still propose d_2. The $risk_1^1 = (5-3)/5 = 0.4$ and $risk_2^1 = (2-1)/2 = 0.5$ so this does also shift the balance of risk, but it yields a lower payoff for agent 2 so d_3 should be preferred as a concession.

You can continue tracing further steps in the protocol until agreement is reached. (Of course this is a contrived example – in particular because it does not only consider pareto optimal deals)

Q4 Consider a situation in which two agents 1 and 2 bid for items a and b. We assume that each agent is allowed to obtain only one item. The agents have the following valuation functions:

$$v_1(\{a\}) = 12$$
$$v_1(\{b\}) = 3$$
$$v_2(\{a\}) = 6$$
$$v_2(\{b\}) = 1$$

1. Which allocation will be assigned by the Vickrey-Clarke-Groves mechanism (VCG Mechanism) if both agents are truthful about their valuations?
2. Calculate the utility each agent will pay to the mechanism.
3. Can the agents benefit by lying about their true valuation functions?

Solution suggestions: First, we overview the VCG mechanism. Every agent declares a valuation function $\hat{v}_i$, which may not be true. The mechanism choses the allocation that maximises the social welfare:

$$Z_1^*, \ldots, Z_n^* = \arg \max_{(Z_1,\ldots,Z_n) \in alloc(\mathcal{Z}, Ag)} sw(Z_1, \ldots, Z_n, \hat{v}_1, \ldots, \hat{v}_i, \ldots, \hat{v}_n)$$

Every agent pays to the mechanism an amount p_i as 'compensation' for the utility other agents lose by i participating:

$$p_i = sw_{-i}(Z_1', \ldots, Z_n', \hat{v}_1, \ldots, v^0, \ldots, \hat{v}_n) - sw_{-i}(Z_1^*, \ldots, Z_n^*, \hat{v}_1, \ldots, \hat{v}_i, \ldots, \hat{v}_n), \text{ where}$$

$$Z_1', \ldots, Z_n' = \arg \max_{(Z_1,\ldots,Z_n) \in alloc(\mathcal{Z}, Ag)} sw(Z_1, \ldots, Z_n, \hat{v}_1, \ldots, v^0, \ldots, \hat{v}_n)$$

$v^0(Z) = 0$ for all $Z \subseteq \mathcal{Z}$ is the 'indifferent' valuation function. $sw_{-i}(Z_1, \ldots, Z_n) = \sum_{j \in Ag: j \neq i} v_j(Z_j)$ is the social welfare of all agents but i.

1. There are two possibly socially optimal allocations: $\{a\}, \{b\}$ and $\{b\}, \{a\}$. The social welfare for these allocations will be: $sw(\{a\}, \{b\}, v_1, v_2) = v_1(\{a\}) + v_2(\{b\}) = 12 + 1 = 13$ and $sw(\{b\}, \{a\}, v_1, v_2) = v_1(\{b\}) + v_2(\{a\}) = 3 + 6 = 9$. The VCG mechanism will chose $\{a\}, \{b\}$, the allocation that maximises the social welfare.
2. In order to determine this for agent 1, we first need to determine the socially optimal allocation if 1 does not participate in the auction. This will allocate $\{a\}$ to agent 2, since 2 prefers a from b. Therefore, $p_1 = sw_{-1}(\{\}, \{b\}, v^0, v_2) - sw_{-1}(\{a\}, \{b\}, v_1, v_2) = v_2(\{a\}) - v_2(\{b\}) = 6 - 1 = 5$. The participation of agent 2 does not affect the items agent 1 is allocated: $p_2 = sw_{-2}(\{a\}, \{\}, v_1, v^0) - sw_{-2}(\{a\}, \{b\}, v_1, v_2) = v_1(\{a\}) - v_1(\{a\}) = 0$.
3. The utility 1 has after the auction if he does not lie about his true preferences is $v(\{a\}) - p_1 = 12 - 5 = 7$. We assume that agent 1 lies and declares function v_1', which is different from v_1. If a is still allocated to 1 and b to 2, the compensation he needs to pay will still be the same, as it is calculated based on the other agent's valuation function. Therefore, there would be no gain to agent 1. We now consider the case in which the lie results in the different allocation in which 1 receives b. The resulting utility agent 1 gains from the auction would be $v_1(\{b\}) - p_1' = v_1(\{b\}) - sw_{-1}(\{\}, \{a\}, v^0, v_2) + sw_{-1}(\{b\}, \{a\}, v_1', v_2) = v_1(\{b\}) - v_2(\{a\}) + v_2(\{a\}) = v_1(\{b\}) = 3$. Note here that the lie has no effect on the calculation of the actual utility gain. Finally, If the lie results in 1 being allocated no objects, then he gains zero utility. Therefore, in any case agent 1 has no incentive to lie.

   The utility 2 gets if he does not lie about his true preferences is $v(\{b\}) - p_2 = 1$. We assume that agent 2 lies and declares function v_2', which is different from v_2. If a is still allocated to 1 and b to 2, the compensation he needs to pay will still be the same, as it is calculated based on the other agent's valuation function. We now consider the case in which the lie results in a different allocation, in which 2 receives a and 1 receives b. The utility agent 2 gains in this case would be $v_2(\{a\}) - p_2' = v_2(\{a\}) - sw_{-2}(\{a\}, \{\}, v_1, v^0) + sw_{-2}(\{b\}, \{a\}, v_1, v_2') = v_2(\{a\}) - v_1(\{a\}) + v_1(\{b\}) = 6 - 12 + 3 = -3$. Finally, if lying results in 2 being allocated nothing, 2 does not receive any utility. Therefore, agent 2 has no incentive to lie.