Agent-Based Systems

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Lecture 11 – Resource Allocation

Where are we?

- Coalition formation
- The core and the Shapley value
- Different representations
- Simple games
- Qualitative coalitional games

Today . . .

- **Resource Allocation**

Auctions

- Auctions = method for allocating scarce resources in a society given preferences of agents
- Most common types of auctions:
  - English (first-price open-cry ascending), Dutch (reverse), first-price sealed bid, Vickrey auction (second-price sealed bid)
- Additional variations depending on following characteristics:
  - private-value, public-value, correlated value auctions
  - risk-neutral, risk-seeking, risk-averse bidders/auctioneer
- Some interesting issues/problems:
  - **Lying** (lying bidders, lying auctioneer)
  - Bidder **collusion**
  - Incentive for **counterspeculation**

The English Auction (EA)

- Each bidder raises freely his bid (in public), auction ends if no bidder is willing to raise his bid anymore
- Bidding process public ➡ in correlated auctions, it can be worthwhile to counterspeculate
- In correlated value auctions, often auctioneer increases price at constant/appropriate rate, also use of reservation prices
- Dominant strategy in private-value EA: bid a small amount above highest current bid until one's own valuation is reached

The English Auction (EA)

- Advantages:
  - Truthful bidding is individually rational & stable
  - Auctioneer cannot lie (whole process is public)
- Disadvantages:
  - Can take long to terminate in correlated/common value auctions
  - Information is given away by bidding in public
  - Use of **shills** (in correlated-value EA) and “minimum price bids” possible, to drive prices
  - Bidder collusion **self-enforcing** (once agreement has been reached, it is safe to participate in a coalition) and identification of partners easily possible

Dutch/First-Price Sealed Bid Auctions

- Dutch (descending) auction: seller continuously lowers prices until one of the bidders accepts the price
- First-price sealed bid: bidders submit bids so that only auctioneer can see them, highest bid wins (only one round of bidding)
- DA/FPSB strategically equivalent (no information given away during auction, highest bid wins)
- Advantages:
  - Efficient in terms of real time (especially Dutch)
  - No information is given away during auction
  - Bidder collusion not self-enforcing, and bidders have to identify each other

Dutch/First-Price Sealed Bid Auctions – Problems

- No dominant strategy, individually optimal strategy depends on assumptions about others’ valuations
- One would normally bid less than own valuation but just enough to win ➡ Incentive to counter-speculate
- Without incentive to bid truthfully, computational resources might be wasted on speculation
- Another problem: lying auctioneer
- Would be nice to combine efficiency of Dutch/FPSB with **incentive compatibility** of English auction ➡ Vickrey auction can be seen as attempt to achieve this

The Vickrey Auction (VA)

- Second-price sealed bid: Highest bidder wins, but pays price of second-highest bid
- Advantages:
  - Truthful bidding is dominant strategy
  - No incentive for counter-speculation
  - Computational efficiency
- Disadvantages:
  - Bidder collusion self-enforcing
  - Lying auctioneer
- Unfortunately, VA is not very popular in real life
- But very successful in computational auction systems

Further issues in auctions

- **Pareto efficiency**: all protocols allocate auction item to the bidder who values it most (in isolated private value/common value auctions)
  - But this result requires risk-neutrality if there is some uncertainty about own valuations
- **Revenue equivalence** in terms of expected revenue among all protocols if valuations independent, bidders risk-neutral and auction is private value
- **Winner's curse** in correlated/common value auctions
  - If I win, I always know I won't get to re-sell at the same price, because others value the goods less!

Further issues in auctions (II)

- Some properties of protocols change
  - if there is uncertainty about own valuations
  - if one can pay to obtain information about others' valuations
  - if we are looking at sequential (multiple) auctions
- Undesirable private information revelation
  - Example: truthful bidding in EA/VA may lead sub-contractors to re-negotiate rates after finding out that price was lower than they thought
- In terms of communication, auctions are not a very expressive method of negotiation
  - Solely concerned with determining a selling price for some item
  - Will look at bargaining and argumentation in next two lectures

Combinatorial Auctions

- Generalised model of resource allocation, auctioning bundles of goods $\mathcal{Z} = \{z_1, \ldots, z_n\}$ instead of single items
- A **valuation function** $v_i : \mathbf{2}^{\mathcal{Z}} \to \mathbb{R}$ indicates how much $Z \subseteq \mathcal{Z}$ is worth to agent i
- Sensible properties of valuation functions:
  - **Normalisation:** $v(\emptyset) = 0$
  - **Free disposal:** $Z_1 \subseteq Z_2$ implies $v(Z_1) \leq v(Z_2)$
- The **outcome** is an allocation $Z_1, Z_2, \ldots, Z_n$ of goods being auctioned among the agents
- **Maximising social welfare:**
  - $Z_1^*, \ldots Z_n^* = \arg\max_{(Z_1,\ldots,Z_n)\in alloc(\mathcal{Z},Ag)} sw(Z_1, \ldots, Z_n, v_1, \ldots, v_n)$
    where $sw(Z_1, \ldots, Z_n, v_1, \ldots, v_n) = \sum_{i=1}^{n} v_i(Z_i)$

Combinatorial Auctions (II)

- **Winner determination:** computing the optimal allocation $Z_1^*, \ldots Z_n^*$ given valuations submitted by bidders
- Prone to strategic manipulation as agents may not reveal their true valuations (e.g. may overstate the value of possible bundles)
- **Representational complexity:** exponential in the number of goods (imagine listing all possible valuations of all bundles)
- **Computational complexity:** winner determination is NP-hard even under restrictive assumptions

Bidding Languages

- As before, we want to have succinct representation schemes for valuation functions
- **Atomic Bid:** $\beta = (Z, p)$, where $Z \subseteq \mathcal{Z}$ and $p \in \mathbb{R}_+$ is the price
- A bundle of goods Z' **satisfies** (Z, p) if $Z \subseteq Z'$
  - Bundle $\{a, b, c\}$ satisfies the atomic bid $(\{a, b\}, 4)$
  - Bundle $\{b, d\}$ does not satisfy the atomic bid $(\{a, b\}, 4)$
- An atomic bid $\beta = (Z, p)$ defines a valuation function v_β

$$v_\beta(Z') = \begin{cases} p & \text{if } Z' \text{ satisfies } (Z, p) \\ 0 & \text{otherwise} \end{cases}$$

- Not sufficient to express any valuation function

XOR bids

- We specify a number of bids, but we will par for at most one
- $\beta = (Z_1, p_1)$ XOR $\cdots$ XOR (Z_k, p_k)

$$v_\beta(Z') = \begin{cases} 0 & \text{if } Z' \text{ does not satisfy any of} \\ & (Z_1, p_1),\ \ldots,\ (Z_k, p_k) \\ \max\{p_i | Z_i \subseteq Z'\} & \text{otherwise} \end{cases}$$

- Example: $\beta = (\{a, b\}, 3)$ XOR $(\{c, d\}, 5)$
  - $v_\beta(\{a\}) = 0$
  - $v_\beta(\{a, b\}) = 3$
  - $v_\beta(\{c, d\}) = 5$
  - $v_\beta(\{a, b, c, d\}) = 5$
- XOR bids are fully expressive, number of bids may be exponential in $|\mathcal{Z}|$, $v_\beta(Z)$ can be computed in polynomial time

OR bids

- Combine more than one atomic statement disjunctively
- $\beta = (Z_1, p_1)$ OR $\cdots$ OR (Z_k, p_k)
- The valuation for $Z' \subseteq \mathcal{Z}$ is determined w.r.t. atomic bids W s.t.:
  - every bid in W is satisfied by Z'
  - each pair of bids in W has mutually disjoint sets of goods
  - there is no other subset of bids W' from W satisfying the first two conditions that $\sum_{(Z_i, p_i) \in W'} p_i > \sum_{(Z_j p_j) \in W} p_j$
- Example: $\beta = (\{a, b\}, 3)$ OR $(\{c, d\}, 5)$
  - $v_\beta(\{a\}) = 0$, $v_\beta(\{a, b\}) = 3$, $v_\beta(\{c, d\}) = 5$, $v_\beta(\{a, b, c, d\}) = 8$
- Not fully expressive, consider:
  - $v(\{a\}) = 1$, $v(\{b\}) = 1$, $v(\{a, b\}) = 1$
- Can be exponentially more succinct than XOR bids

The VCG Mechanism (I)

- Terminology:
  - 'Indifferent' valuation function: $v^0(Z) = 0$ for all $Z \subseteq \mathcal{Z}$
  - $sw_{-i}(Z_1, \ldots, Z_n) = \sum_{j \in Ag: j \neq i} v_j(Z_j)$, social welfare of all agents but i
- The **Vickrey-Clarke-Groves mechanism** (VCG Mechanism):
  1. Every agent declares a valuation function $\hat{v}_i$ (may not be true)
  2. Mechanism choses the allocation that maximises the social welfare:

$$Z_1^*, \ldots, Z_n^* = \arg \max_{(Z_1, \ldots, Z_n) \in alloc(\mathcal{Z}, Ag)} sw(Z_1, \ldots, Z_n, \hat{v}_1, \ldots, \hat{v}_i, \ldots, \hat{v}_n)$$

  3. Every agent pays to the mechanism an amount p_i ('compensation' for the utility other agents lose by i participating)

$$p_i = sw_{-i}(Z_1', \ldots, Z_n', \hat{v}_1, \ldots, v^0, \ldots, \hat{v}_n) - sw_{-i}(Z_1^*, \ldots, Z_n^*, \hat{v}_1, \ldots, \hat{v}_i, \ldots, \hat{v}_n), \text{ where}$$

$$Z_1', \ldots, Z_n' = \arg \max_{(Z_1, \ldots, Z_n) \in alloc(\mathcal{Z}, Ag)} sw(Z_1, \ldots, Z_n, \hat{v}_1, \ldots, v^0, \ldots, \hat{v}_n)$$

The VCG Mechanism (II)

- The VCG mechanism is **incentive compatible:**
  - telling the truth is the dominant strategy
- Generalisation of the Vickrey auction: for a single good VCG reduces to the Vickrey mechanism
  - p_i would be the amount of the second highest valuation
- **Shows that social welfare maximisation can be implemented in dominant strategies in combinatorial auctions**
- Computing VCG payments is NP-hard

Summary

- Different auction types and properties
- Combinatorial Auctions
- Bidding Languages
- The VCG mechanism
- Next time: **Bargaining**