The GAMES project
Games and Automata for Synthesis and Verification

Julian Bradfield

The GAMES project

GAMES was a Research and Training Network in the Fifth Framework. The participating institutions are RWTH Aachen (coordinator), U. Bordeaux 1, U. Edinburgh, U. Paris 7, Rice U., Uppsala U., T.U. Vienna and U. Warsaw.

Objectives: There is a growing need for formal methods that guarantee the reliability, correctness, and efficiency of computerised systems. This project addresses this challenge by developing specification and validation methodologies that are based on games and automata. Oriented at both foundational research and modern applications, this network aims to provide a novel set of techniques for the synthesis and validation of computing systems.

Areas of research

GAMES notionally classified its work into nine tasks: Minimizing automata and simplifying games; Algorithmic analysis of parity games; Synthesis and testing for reactive computation; Analysis techniques for infinite-state systems; Linear-time model checking; Game models for protocols; Logics, games, and efficient query evaluation; Automata and query languages for semistructured data; Accessibility and dissemination.

Of course, most work in the project touched on several of the tasks.

Outputs

The GAMES project has around 180 publications, mostly conference and journal papers. As a Research and Training Network, one of the main deliverables was the employment of pre- and post-docs, who had to move country to be eligible. The network delivered more than the contracted number of person-months, and the feedback to the European Commission from the researchers indicates that their time with the project was valuable both intellectually and for future employment.

GAMES at Edinburgh

Faculty involved were Julian Bradfield (site leader), Kousha Etessami, Perdita Stevens, Colin Stirling. Students involved were Jan Obdržálek and Jennifer Tenzer. Funded researchers were Stephan Kreutzer and Sandra Quickert (post-doc), and Michael Ummels and Magnus Johansson (pre-doc).

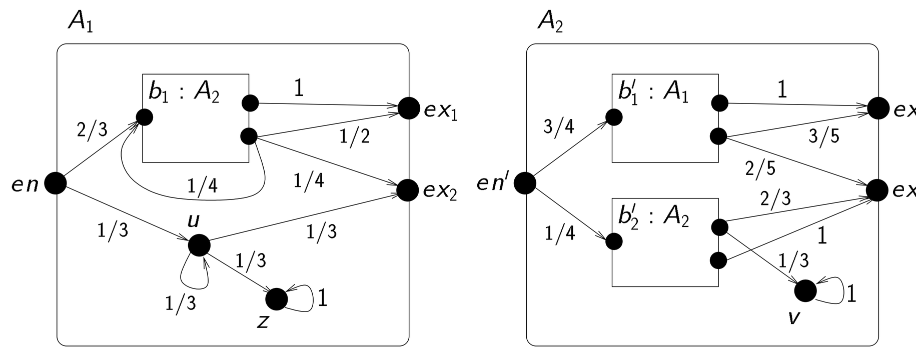
Further information

The main project web site is `http://www.games.rwth-aachen.de/`, and the local contact is Julian Bradfield `jcb@inf.ed.ac.uk`.

Recursive Markov Processes

Work by Etessami, together with Mihalis Yannakakis (Columbia), is among the most significant results in the project.

The classical Markov chain is a (usually) finite state system with probabilistic transitions between states. *Recursive Markov Chains* allow 'subroutine call' between Markov chains: a transition into the calling state is identified with a transition to the initial state of the callee, and transitions out of the calling state is identified with transitions from the exit state(s) of the callee. This defines a class of *infinite state stochastic* systems with many applications in modelling. Similarly, the classical Markov Decision Processes of control theory extend to Recursive MDPs.



Several major theoretical results have been obtained. One general slogan is, "*qualitative* analysis (e.g. 'does something bad/good happen almost surely?') for recursive MCs is at least as hard as *quantitative* analysis (e.g. 'with what probability does something bad/good happen?') for ordinary MCs". A technically interesting result is that the square root sum problem, known to be in PSpace but not known to be in NP, reduces to quantitative termination for RMCs where only one exit state is allowed, and to qualitative termination for general RMCs.

An algorithmic breakthrough in this work is the proof that Newton's method is guaranteed to converge monotonically on a wide class of non-linear equational systems; and it converges fast in practice. This has several practically important applications – for example, the stochastic context-free grammars used in Natural Language Processing are in this class, and can now be computed with much faster.

Further results have been obtained on verifying properties of RMCs expressed in Linear Temporal Logic or Büchi automata, which is typically hard (PSpace) for general RMCs, but polynomial for restricted classes. However, even for simple RMDPs, LTL model-checking is undecidable.

For more information on these results, see
`http://homepages.inf.ed.ac.uk/kousha/`
or contact Kousha Etessami `kousha@inf.ed.ac.uk`.

Complexity of IF temporal logics

Bradfield and Kreutzer found surprising results on the 'independence-friendly mu-calculus' introduced by Bradfield a couple of years earlier.

Hintikka's 'independence-friendly' (IF) logic allows an inner quantifier $\exists y/x$ to be independent of an outer quantifier $\forall x$, so that the choice of witness for y must be made uniformly in x. This allows the expression of the well-known Henkin quantifier $\begin{smallmatrix}\forall x \exists y\\ \forall u \exists v\end{smallmatrix}$, and so goes well beyond first-order logic (to existential second order). IF logic is interesting from a CS viewpoint because it provides a natural logic for some forms of concurrency. In terms of games, it corresponds to games of *imperfect information*, where one player does not know (some of) the previous choices of the other.

Modal mu-calculus, or modal fixed point logic, is one of the core logics of the GAMES project. By combining fixed point operators (effectively finite and infinite looping constructs) with modal operators ('in some next state . . . '), it provides a logic encompassing most other temporal logics, and corresponding exactly to 'parity automata' and 'parity games'. It is also a logic with limited second-order power.

We showed that the combination of IF concepts with mu-calculus, itself a slightly delicate task, results in a surprising jump in expressive power. The resulting logic is at least ExpSpace-hard to verify (combined complexity), captures ExpTime, and on finite structures it is so strong that it includes all of second-order logic.

Transfinite mu-calculus

Bradfield and Quickert, together with Jacques Duparc (Lausanne), solved a problem posed by Bradfield several years ago. It had been shown that there was a natural and elegant connection, via the 'game quantifier', between the hierarchy of 'fixpoint alternation' in mu-calculus, which is a hierarchy of increasing expressive power, and a hierarchy of 'difference sets' well known in descriptive set theory. The question was whether this connection could be meaningfully extended to transfinite levels of the hierarchy, a result which might give an interesting new piece of information about an old set-theoretic issue, the power of the game quantifier.

After dealing with a number of technical difficulties, which had obstructed previous attempts, the extended result was obtained in the form hoped for, and the desired set-theoretic consequence obtained: we now have a comprehensible description of games up to Δ^0_3 in their winning condition complexity.

In addition, we were able to show a highly technical but ultimately neat analysis of the hierarchies in terms of transfinite arithmetic operations on 'Wadge degrees', another well-known set-theoretic concept.

THE UNIVERSITY OF EDINBURGH

School of informatics