Inconstancy: An Ontology Repair Plan for Adding Hidden Variables

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Abstract

We describe mechanisms for automated evolution of ontologies to adapt to new circumstances and to make them better suited to the given task. If a conflict is detected between the original theory and new experimental evidence, a repair is required to resolve the inconsistency and to recover from failure. The rules for conflict diagnosis and transformation of the ontologies are composed together into *ontology repair plans*. The repair plans have been implemented in the GALILEO system and successfully evaluated on a diverse range of examples from the history of physics. By applying the described repair plans, the initially incorrect physical theories in our examples are repaired to become consistent with experimental results.

Introduction

For different software agents to properly communicate, they must align their ontologies so that they can correctly understand semantically related concepts with different representations. When a large, diverse and evolving community of autonomous agents are continually engaged in online negotiations, e.g., on the Semantic Web, it is not practical to manually pre-align the ontologies of all agent pairs – it must be done dynamically and automatically. Furthermore, persistent agents must be able to cope with a changing world and changing goals. This requires evolving their ontologies as their problem solving task evolves; the W3C call this *ontology evolution*[1]. In evolving a logical theory, it is not always enough just to perform belief revision. Often it is necessary to change the underlying signature of the theory, e.g., to add, remove or alter the functions, predicates, types, etc. of the theory.

The automation of reasoning as deduction in logical theories, including ontologies, is well established. Such logical theories are usually inherited from the literature or are built manually for a particular reasoning task. They are then regarded as fixed, but we will argue that they should be regarded as fluid. As Pólya and others have argued, appropriate representation is the key to successful problem solving (Pólya 1945). It follows that a successful problem solver must be able to choose or construct the representation best suited to solving the current problem. Some of the most seminal episodes in human problem solving, especially those in scientific discoveries, required radical representational change.

By automating signature evolution in logical theories, agents can autonomously resolve conflicts in signatures and representations in their ontologies. To this end, we are developing a collection of *ontology repair plans*, which agents can execute to repair their knowledge representations. Repair plans are composed of trigger patterns that help detect contradictions and transform old ontologies containing conflicting theories into new consistent ones.

We are now applying our techniques in the domain of physics, as described in (Bundy 2007) and (Bundy and Chan 2008). This is an excellent domain as many of its seminal advances throughout its history involve changing the way physicists view the world, which can actually be seen as ontology evolution. The two repair plans so far developed, which we call "*Where's my stuff?*" (WMS) and *Inconstancy*, have been implemented in the GALILEO (Guided Analysis of Logical Inconsistencies Leads to Evolved Ontologies) system using a higher-order logic (HOL) programming language, λProlog (Miller and Nadathur 1988). HOL is useful to us because higher-order ontologies appear to be required in the physics domain, since many of the concepts, e.g., calculus, are essentially higher-order. Later in the paper, we will show how some physical concepts, such as orbits and trajectories, can be represented as graphs objects.

To help us develop and evaluate the repair plans, we have applied the implementations of WMS and Inconstancy to numerous examples from various areas of physics. For instance, WMS has been successfully applied to the discovery of latent heat, an apparent paradox of lost energy of a bouncing ball, the speculation of an additional planet Vulcan, and the invention of dark matter. Similarly, Inconstancy has been applied to the generalisation of the Gravitational Constant, the exten-

[1] `http://www.w3.org/TR/webont-req/#goal-evolution`

sion of Boyle's Law to the Gas Law, the measurement of the travel time of light, and the transition from the 2- to 3-body planetary problem. The focus of this paper is on the development and evaluation of the Inconstancy repair plan, so all of the mentioned applications of Inconstancy are described. For more details on WMS, see (Bundy 2007).

Aims, Hypothesis, Objectives and Evaluation

The *aim* of our project is to demonstrate that automatic ontology evolution via repair plans is computationally feasible and can account for the kinds of ontology evolution that are observed in human problem solving. The specific *hypothesis* that will be evaluated in the project is that:

> *A few generic, ontology repair plans can account for a large number of historical instances of ontology evolution in the physics domain*

We plan to support this hypothesis by achieving the following *objectives*:

1. Informed by a development set of examples of ontology evolution in physics, we will construct between five and ten generic repair plans.
2. We will develop generic ontology evolutionary machinery that can use these repair plans to diagnose faulty ontologies and use this diagnosis to repair the ontologies to remove the faults.
3. These repair plans and the evolution machinery will be implemented as HOL programs and evaluated on a test set of examples of ontology evolution in physics, and possibly other relevant domains, e.g., maths, other sciences.

We will isolate and generalise the atomic ontology repair operations arising in our case studies. Also, since repairs need to be minimal in order to avoid unmotivated and unnecessary repairs, then a suitable concept of *minimality* needs to be defined and our repairs shown to be minimal with respect to it. Figure 1 outlines an approach we are currently exploring.

The *evaluation* of the repair plans will assess to what extent they (a) create a new ontology that escapes the failures diagnosed in the prior ontology and (b) to what extent this emulates the historical process of ontology evolution. In assessing (b), we will take a normative stance, i.e., we are not interested in exactly modelling the historical process, with all its idiosyncrasies, false starts, coincidences, etc. Rather, we will be content with capturing a 'rational reconstruction' of that history. Since we expect the size of the test set for each repair plan to be measured in tens rather than hundreds or thousands, a quantitative or statistical analysis will be inappropriate. Rather, our evaluation methodology will be based on discursive analysis of a series of case studies. We will be looking specifically for generality and explanatory power from our repair plans, so will seek diversity in our test set and emergent abstraction from the uniform processing of apparently diverse examples.

To define a concept of minimal repair we are experimenting with extending *conservative extension* to changes of signature, namely:

$$\phi \in Sig(O) \implies (\nu(O) \vdash \nu(\phi) \iff O \vdash \phi)$$

where $\phi \in Sig(O)$ means that ϕ is a formula in the signature of ontology O and $\nu(\phi)$ is the repaired ϕ in repaired ontology $\nu(O)$. In the WMS repair plan, both $\nu(O_t)$ and $\nu(O_s)$ are conservative in this extended sense. Their combination, of course, is not, since the purpose of the repair is to prevent a contradiction being derived in the repaired combination.

Figure 1: Minimal Ontology Repair via Conservative Extensions

Related Work

We first investigated the automation of signature evolution in ORS (Ontology Repair System): an automated system for repairing faulty ontologies in response to unexpected failures when executing multi-agent plans (McNeill and Bundy 2007). ORS evolved first-order ontologies by first diagnosing their faults via the execution failures of multi-agent plans, then using this diagnosis to guide repairs to these ontologies. These repairs were not mere belief revisions, but changes to the underlying signatures, e.g., adding or removing function arguments, and splitting or conflating functions. These signature changes include but go beyond mere definitional extensions.

There is also related work on repairing inconsistencies in OWL ontologies, for instance (Kalyanpur et al. 2006). It focuses on strategies for removing inconsistent axioms and for identifying syntactical modelling errors in OWL ontologies to assist users to rewrite faulty axioms. Our focus, in contrast, is on repairing deeper conceptual errors in the underlying physical theory, rather than fixing errors in the use of the OWL operators. In addition, our goal is not to provide assistance to users, but to automate the repair process.

Ontology Repair Plans

The two repair plans so far developed, WMS and Inconstancy, roughly correspond to the refinement operations of splitting a function and adding an argument, respectively. We have found multiple examples of these repairs across the history of physics, but are always interested in additional ones.

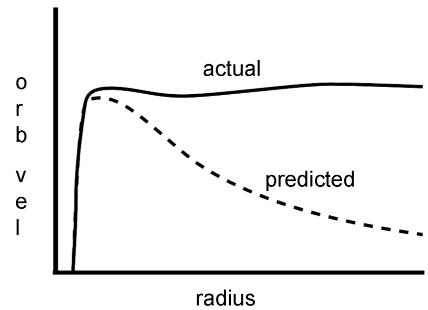
The WMS repair plan aims at resolving contradictions arising when the predicted value returned by a function does not match the observed value. This is modelled by having two theories, corresponding to the prediction and the observation, with different values for this function. To break the inconsistency, the conflicting function is split into three new functions: *visible*, *invisible* and *total*. The conflicting function becomes the total function in the predictive theory and the visible function in the observation theory[2]. The invisible function is defined as the difference between them, and this new definition is added to the predictive theory. The intuition behind this repair is that the discrepancy arose because the function was not being applied to the same *stuff* in the predictive and the observational theories – the visible stuff was observed but the invisible stuff was not.

An example application of WMS is to repair an initially incorrect theory about orbital velocities of stars by introducing *dark matter*. The evidence for dark matter arises from various sources, for instance, from an anomaly in the orbital velocities of stars in spiral galaxies identified by Rubin (Rubin, Thonnard, and Ford 1980). Given the observed distribution of mass in these galaxies, we can use Newtonian Mechanics to predict that the orbital velocity of each star should be inversely proportional to the square root of its distance from the galactic centre (called its *radius*). However, observations of these stars show their orbital velocities to be roughly constant and independent of their radius. Figure 2 illustrates the predicted and actual graphs. In order to account for this discrepancy, it is hypothesised that galaxies also contain a halo of, so called, dark matter, which is invisible to our radiation detectors, because it does not radiate, so can only be measured indirectly. By applying the principle of WMS, the whole galaxy can be represented by the total function, made up of visible and invisible parts. Since the visible part is defined in the predictive theory, it represents the observable part of the galaxy, whereas the invisible part represents dark matter.

The Inconstancy repair plan is triggered when there is a conflict between the predicted independence and the observed dependence of a function on some parameter, i.e., the observed value of a function unexpectedly varies when it is predicted to remain constant. This generally requires several observational theories, each with different observed values of the function, as opposed to the one observational theory in the WMS plan. To effect the repair, the parameter causing the unexpected variation is first identified and a new definition for the conflicting function is created that includes this new parameter. The nature of the dependence is induced from the observations using curve-fitting techniques.

In the next section, we provide a revised formulation of the Inconstancy given in (Bundy and Chan 2008). The major difference between the old and the revised is in one of the trigger formulae used to detect a variation in the observed values.

[2]There are situations in which these roles are inverted (Bundy 2007)



This diagram is taken from `http://en.wikipedia.org/wiki/Galaxy_rotation_problem`. *The x-axis is the radii of the stars and the y-axis is their orbital velocities. The dotted line represents the predicted graph and the solid line is the actual graph that is observed.*

Figure 2: Predicted *vs* Observed Stellar Orbital Velocities

The Inconstancy Ontology Repair Plan

Suppose we have an ontology O_t representing the current state of a physical theory and some ontologies O_s representing sensory information arising from experiments, such that different sensory ontologies give distinct values for function $stuff(\vec{s_i})$ in different circumstances. Suppose function $V(\vec{s_i}, \vec{b_i})$ of the i^{th} sensory ontology, where $\vec{b_i}$ contains variables distinguishing among these circumstances, returns distinct values in each of these circumstances, but is *not* one of the parameters in $\vec{s_i}$, i.e., $stuff(\vec{s_i})$ does not depend on $V(\vec{s_i}, \vec{b_i})$. We will call $stuff(\vec{s_i})$ the *inconstancy* and $V(\vec{s_i}, \vec{b_i})$ the *variad*. The Inconstancy repair plan establishes a relationship between the variad $V(\vec{s_i}, \vec{b_i})$ and the inconstancy $stuff(\vec{s_i})$. The inconstancy might, for instance, be the gravitational constant G and the variad might be the acceleration of an orbiting star due to the gravity, which is suggested by MOdified Newtonian Dynamics (MOND).

Trigger: If $stuff(\vec{s_i})$ is measured to take different values in different circumstances, then the following trigger formulae will be matched.

$$\begin{array}{rcl} O_s(V(\vec{s_1}, \vec{b_1}) = v_1 \ldots) & \vdash & stuff(\vec{s_1}) = c_1 \\ \vdots & \vdots & \\ O_s(V(\vec{s_n}, \vec{b_n}) = v_n \ldots) & \vdash & stuff(\vec{s_n}) = c_n \end{array} \quad (1)$$

$$O_t \vdash stuff(\vec{x}) ::= c(\vec{x}) \quad (2)$$

$$\exists i \neq j.\ O_t \vdash stuff(\vec{s_i}) - c_i \neq stuff(\vec{s_j}) - c_j \quad (3)$$

where $\vec{x}$ can be instantiated to $\vec{s_i}$ for $1 \leq i \leq n$, $O_s(V(\vec{s_i}, \vec{b_i}) = v_i)$ is the sensory ontology containing observations made under the condition that

$V(\vec{s_i}, \vec{b_i}) = v_i$ and $V(\vec{s_i}, \vec{b_i})$ is not an existing argument of $\mathit{stuff}(\vec{s_i})$, i.e., $V(\vec{s_i}, \vec{b_i}) \notin \vec{s_i}$.

Add Variad: The repair is to change the signature of all the ontologies to relate the inconstancy, $\mathit{stuff}(\vec{x})$, to the variad, $V(\vec{x}, \vec{y})$:

$$\nu(\mathit{stuff}) ::= \lambda \vec{y}, \vec{x}.\ F(c(\vec{x}), V(\vec{x}, \vec{y})) \tag{4}$$

where F is a new function, whose value we will seek to determine by curve fitting against the data from the sensory ontologies.

Create New Axioms: We calculate the axioms of the new ontologies in terms of those of the old as follows:

$$\begin{aligned} Ax(\nu(O_s(V(\vec{s_i}, \\ \vec{b_i}) = v_i \ldots))) \quad &::= \quad \{\phi\{\mathit{stuff}/\nu(\mathit{stuff})(\vec{b_i})\} \mid \\ & \qquad \phi \in Ax(O_s(V(\vec{s_i}, \vec{b_i}) = v_i))\} \\ Ax(\nu(O_t)) \quad &::= \quad \{\phi\{\mathit{stuff}/\nu(\mathit{stuff})(\vec{y})\} \mid \\ & \qquad \phi \in Ax(O_t) \setminus \{\mathit{stuff}(\vec{x}) ::= \\ & \qquad c(\vec{x})\}\} \cup \{\nu(\mathit{stuff}) ::= \\ & \qquad \lambda \vec{y}, \vec{x}.\ F(c(\vec{x}), V(\vec{x}, \vec{y}))\} \end{aligned}$$

i.e., the axioms of $\nu(O_t)$ and the $\nu(O_s(V(\vec{s_i}, \vec{b_i}) = v_i))$ are the same as for O_t and $O_s(V(\vec{s_i}, \vec{b_i}) = v_i \ldots)$ except for the replacement of the old *stuff* with $\nu(\mathit{stuff})$ and the replacement of the definition of $\mathit{stuff}(\vec{x})$ by the definition of $\nu(\mathit{stuff}(\vec{x}))$ in $\nu(O_t)$.

To discover the meaning of the function F, we follow the tradition of Langley's BACON program (Langley et al. 1983) by using curve fitting. The $O_s(V(\vec{s_i}, \vec{b_i}) = v_i \ldots)$ ontologies provide a useful collection of equations: $F(c(\vec{s_i}), V(\vec{s_i}, \vec{b_i})) = c_i$ for $i = 1, \ldots, n$. Curve fitting techniques, e.g., regression analysis, can be applied to these equations to approximate a definition of F. This hypothesis can then be tested by creating additional observations $O_s(V(\vec{s_j}, \vec{b_j}) = v_j \ldots)$, for new values of $V(\vec{s_j}, \vec{b_j})$, and confirming or refuting the hypothesis. Furthermore, a noteworthy aspect is that missing arguments (variads) can be considered as missing *causes.*

Discussion on the Revised Trigger

Unlike (3), the corresponding trigger formula described in (Bundy and Chan 2008) directly compares two observed values, c_i and c_j, i.e.

$$\exists i \neq j.\ O_t \vdash c_i \neq c_j \tag{5}$$

where O_t is the theoretical ontology. There is a need for revision because an intuition behind Inconstancy is to trigger the repair if the observed values *unexpectedly* vary. To determine whether a variation is indeed unexpected, it should be compared to the original expectations. That said, (5) does not account for the original expectations of the conflicting function *stuff* and could incorrectly trigger the repair plan to produce both false-positive and false-negative results. For instance, if *stuff* originally depends on some parameter p, i.e., its vector of arguments $\vec{x}$ contains p and $\vec{s_i}$ contains an instantiation of p, then some of the observed values of *stuff* may be different if the conditions under which the values are observed correspond to those instantiations of p that give different $\mathit{stuff}(\vec{s_i})$. Of course, an assumption essential for the old trigger is that all parameters beside the variad must be held fixed in each sensory ontology. In many physics problems, however, some parameters cannot naturally be held fixed, e.g., the current time moment. If p cannot be held fixed, then we cannot determine whether the observed variation in *stuff* is caused by the variation in p or in the selected variad. Another example is when the returned values of *stuff* are initially expected to vary, e.g., with time, but changes to the variad cause the observed values of *stuff* to become *constant.* Due to the constancy of the observed values, the repair would not be triggered even the ontologies require repair.

Formula (5) can be adjusted so that it does not directly compare two observed values, c_i and c_j, but instead it compares the *differences* between those values and the expected returned values of $\mathit{stuff}(\vec{s_i})$ and $\mathit{stuff}(\vec{s_j})$:

$$\mathit{stuff}(\vec{s_i}) - c_i \neq \mathit{stuff}(\vec{s_j}) - c_j$$

where $\vec{s_i}$ and $\vec{s_j}$ are instantiations of the arguments of *stuff* in the i^{th} and j^{th} sensory ontologies.

A noteworthy aspect is that the old trigger using (5) is a special case of (1), (2) and (3) by letting $\vec{s_i} = \vec{s_j}$ and $c(\vec{s_i}) = c(\vec{s_j}) = 0$. In other words, $\vec{x}$ is limited to having the same instantiation in each sensory ontology. More importantly, $\mathit{stuff}(\vec{x})$ is implicitly expected to return 0, even though $\mathit{stuff}(\vec{x})$ may have a different definition in O_t. This conflict also proves that there is a need for revision of the trigger.

Application to Modified Newtonian Dynamics

Another explanation of the anomaly in orbital velocities of stars in spiral galaxies depicted in Figure 2 is provided by MOdified Newtonian Dynamics (MOND), proposed by Moti Milgrom in 1981 as an alternative to the dark matter explanation. We have already discussed that dark matter is an example of the WMS plan. Now MOND is an example of the Inconstancy plan. This is a good example of how the same observational discrepancies can trigger different repair plans. Essentially, MOND suggests that the gravitational constant is not a constant, but depends on the acceleration between the objects on which it is acting[3]. It is constant until the acceleration becomes very small and then it depends on this acceleration, which is the case for stars in spiral

[3]It can also be presented as breaking of the equivalence of inertial and gravitational mass, but the varying gravity story fits our purposes better.

galaxies. So, the gravitational constant G can be repaired by giving it an additional argument to become $G(Acc(s))$, where $Acc(s)$ is the acceleration of a star s due to the gravitational attraction between the star and the galaxy in which it belongs. $V(Acc(s))$ is the variad and G is the inconstancy.

We know from the law of universal gravitation that the square of the radius is inversely proportional to the acceleration of the orbiting star due to gravity, with the product of the gravitational constant and the mass of the galaxy being the constant of proportionality, i.e., $Rad(S_i)^2 = \frac{G \times M}{Acc(S_i)}$, where G, M, and $Acc(S_i)$ denote the gravitational constant, the mass of the galaxy, and the acceleration of the star w.r.t. the galaxy in which it belongs. So, we need to collect evidence for a variety of stars: S_i for $1 \leq i \leq n$, where $Acc(S_i)$ varies from large, i.e., S_i is near the centre of the galaxy, to small, i.e., S_i is on the periphery of the spiral galaxy.

The trigger formulae for the Inconstancy plan will then be:

$$
\begin{aligned}
O_s(Acc(S_1) = \\
A_1 \ldots) \quad &\vdash \quad G = M2OV^{-1}(OV(S_1), Mass(\\
&\qquad S_1), \lambda s \in Spiral \setminus \{S_1\}.\\
&\qquad \langle Posn(s), Mass(s)\rangle) \ (= G_1)\\
\vdots \quad & \quad \vdots\\
O_s(Acc(S_n) = \\
A_n \ldots) \quad &\vdash \quad G = M2OV^{-1}(OV(S_n), Mass(\\
&\qquad S_n), \lambda s \in Spiral \setminus \{S_n\}.\\
&\qquad \langle Posn(s), Mass(s)\rangle) \ (= G_n)\\
O_t \quad &\vdash \quad G ::= 6.67 \times 10^{-11}\\
\exists i \neq j.\ O_t \quad &\vdash \quad G - G_i \neq G - G_j
\end{aligned}
$$

where $M2OV^{-1}$ is the inverse of $M2OV$, which takes the value of G, the mass of a star s $Mass(s)$ and the mass distribution of all the remaining stars in the spiral galaxy based on their positions $Posn(s)$, and calculates the orbital velocity of s. $M2OV^{-1}$, therefore, takes the observed orbital velocity of a star s $OV(s)$, the mass of s and the mass distribution of all the remaining stars in the galaxy and calculates what value of G would account for the observed orbital velocity of s.

The formulae above triggers the plan with the following substitution:

$$
\{G/stuff,\ \langle\rangle/\vec{s_i},\ \langle\rangle/\vec{s_j},\ \langle\rangle/\vec{x},\ 6.67 \times 10^{-11}/c,\\
Acc/V,\ \langle S_i\rangle/\vec{b_i}, G_1/c_1,\ G_n/c_n\}
$$

Since G is a constant, $\vec{s_i}$, $\vec{s_j}$ and $\vec{x}$ are simply empty vectors.

Following the instructions for repair, the variad is given to the inconstancy by:

$$
\nu(G) ::= \lambda s.F(6.67 \times 10^{-11}, Acc(s))
$$

and the repaired triggering formulae are therefore:

$$
\begin{aligned}
&\nu(O_s(Acc(&& && (6)\\
&S_1) = A_1 \ldots)) &&\vdash \quad \nu(G)(S_1) = M2OV^{-1}(OV(S_1),\\
& && \qquad Mass(S_1), \lambda s \in Spiral \setminus \{S_1\}.\\
& && \qquad \langle Posn(s), Mass(s)\rangle) \ (= G_1)\\
&\quad\vdots && \qquad \vdots\\
&\nu(O_s(Acc(&& && (7)\\
&S_n) = A_n \ldots)) &&\vdash \quad \nu(G)(S_n) = M2OV^{-1}(OV(S_n),\\
& && \qquad Mass(S_n), \lambda s \in Spiral \setminus \{S_n\}.\\
& && \qquad \langle Posn(s), Mass(s)\rangle) \ (= G_n)\\
&\nu(O_t) &&\vdash \quad \nu(G) ::= \lambda s.F(6.67 \times 10^{-11},\\
& && \qquad Acc(s))
\end{aligned}
$$

which breaks the derivation of the detected contradiction, as required.

The function F can be determined by finding the best-fit curve for the whole dataset, in which each data point represents an observed G_i made under a particular condition $Acc(S_i) = A_i$. F is a reasonable approximation only if a fairly large number of observations of G_i for a wide range of accelerations of stars $Acc(S_i)$ are analysed. If F is a correct and complete approximation of $\nu(G)$, then $F(6.67 \times 10^{-11}, Acc(s))$ returns the unrepaired value 6.67×10^{-11} if a star s has an acceleration much greater than $1.2 \times 10^{-10} ms^{-2}$ (close to the centre of the galaxy). If s has an acceleration that is much less than $1.2 \times 10^{-10} ms^{-2}$ (near the periphery of the galaxy), the value returned will be greater than 6.67×10^{-11} and proportional to $Acc(s)^2 \times Rad(s)^2$, where $Rad(s)$ is the radius of the star's orbit.

Clearly, the repair performed to give (6) and (7) is very different to that performed to give the dark matter theory – respectively, the acceleration of a star becomes a dependent variable of the repaired definition of the Gravitational Constant and the applicability of the repaired Newtonian theory is limited to only the observable part of the galaxy.

Application to the Travel Time of Light

One of the earliest recorded discussions of the speed of light was by Aristotle, who believed that light travelled instantaneously and rejected theories about finite speeds of light. In 1676, a Danish astronomer, Ole Roemer, measured the speed by studying Io, one of Jupiter's moons, which was known to be eclipsed by Jupiter at regular intervals (Ellis and Uzan 2005). Roemer discovered that the eclipses increasingly lagged behind the predicted times, but then started to pick up again. This discovery helped him come up with the theory that when Jupiter and Earth were further apart, there was more distance for light reflecting off Io to travel to Earth and therefore it took longer to reach his telescope.

We can model how Aristotle's (wrong) theory that light appeared instantaneously can be repaired by Roemer's observations of variations in the occurrence times between eclipses of Io, as seen from Earth, to produce the correct theory that light has a finite speed. Let $Time(Light)$ be the time required for light to travel. By Aristotle's theory, the time light requires to travel over any distance is zero because it is instantaneous, i.e. $Time(Light) ::= 0$, which is asserted in the original theoretical ontology, O_t. Let $O_s(Dist(p_1, p_2) = d \ldots)$ be the sensory ontology describing the situation in which the distance light travels between points p_1 and p_2 is d; in this example, D is the distance between Io and the Earth. We would like to collect observations over a reasonable range of such distances between a series of two points, P_i and P_{i+1}, where $1 \leq i \leq n$ such that $Dist(P_i, P_{i+1})$ varies within the range. The trigger formulae of the Inconstancy plan can be matched by the following:

$$\begin{array}{rcl} O_s(Dist(P_1, P_2) = & & \\ D_1 \ldots) & \vdash & Time(Light) = T_1 \\ \vdots & & \vdots \\ O_s(Dist(P_{n-1}, P_n) = & & \\ D_n \ldots) & \vdash & Time(Light) = T_n \\ O_t & \vdash & Time(Light) ::= 0 \\ \exists i \neq j.\, O_t & \vdash & Time(Light) - T_i \neq \\ & & Time(Light) - T_j \end{array}$$

These formulae instantiate the trigger formulae with the following substitution:

$$\{Time(Light)/stuff,\ \langle\rangle/\vec{s_i},\ \langle\rangle/\vec{s_j},\ \langle\rangle/\vec{x},\ 0/c,$$
$$Dist/V,\ \langle P_i, P_{i+1}\rangle/\vec{b_i},\ T_1/c_1, T_n/c_n\}$$

To insert the variad, $Dist(Light, T_i)$, into the new definition, we follow (4) to get:

$$\nu(Time(Light)) ::= \lambda p_1, p_2.\ F(0, Dist(p_1, p_2))$$

The repaired ontologies are therefore:

$$\begin{array}{rcl} \nu(O_s(Dist(P_1, & & \\ P_2) = D_1 \ldots) & \vdash & (\nu(Time(Light))(P_1))(P_2) = D_1 \\ \vdots & & \vdots \\ \nu(O_s(Dist(P_{n-1}, & & \\ P_n) = D_n \ldots) & \vdash & (\nu(Time(Light))(P_{n-1}))(P_n) \\ & & = D_n \\ \nu(O_t) & \vdash & \nu(Time(Light)) ::= \\ & & \lambda p_1, p_2.\ F(0, Dist(p_1, p_2)) \end{array}$$

which together resolve the detected contradiction. The new definition of $Time(Light)$, $\nu(Time(Light))$, is what is required by the general definition of time of travel, which can be put simply that if an object, including a stream of photons, travels at a finite speed, then the time of travel depends on the distance, i.e., $Time \propto Distance$.

Application to the Gas Laws

An example from introductory physics is Boyle's law, which formulates a relationship between the pressure and the volume of a gas. The law was discovered by Robert Boyle in 1662 and it states that given a fixed amount of gas, the pressure P and the volume V of the gas are inversely proportional to each other, i.e., $P \times V = k$, where k is a constant for a particular quantity of gas. Boyle's Law is most famous for being the basis of derivation for the ideal gas law, which provides a complete formulation of the relationship between P, V and the temperature T. We will describe how the Inconstancy plan could use such an observation to modify Boyle's law to resemble the ideal gas law, by diagnosing this dependency on T and adopting it as the variad.

Similar to the previous example, we want to repair the inconstancy's inconsistent dependence on the variad. It is important to note that Boyle's law is a correct account of the relationship between pressure and volume at a fixed temperature and amount of gas, but the law itself is simply incomplete.

We can model the scenario as follows. Let $Pres(gas, t)$, $Vol(gas, t)$, $Temp(gas, t)$ be the pressure, volume and temperature of some gas at moment t. In the original theoretical ontology O_t, assert Boyle's Law as the axiom $Boyle(gas) ::= Pres(gas, t) \times Vol(gas, t)$. Note that $Boyle(gas)$ depends on gas but not on t. Let $O_s(Temp(gas, t) = k \ldots)$ be the sensory ontology describing the situation when the temperature of gas at moment t is k. We need to collect evidence for different temperatures over a range of time moments: T_i for $1 \leq i \leq n$, where $Temp(Gas, T_i)$ varies. We can then match the trigger formulae of the Inconstancy plan with the following formulae:

$$\begin{array}{rcl} O_s(Temp(Gas, & & \\ T_1) = T_1 \ldots) & \vdash & Boyle(Gas) = Pres(Gas, T_1) \times \\ & & Vol(Gas, T_1)\ (= K_1) \\ \vdots & & \vdots \\ O_s(Temp(Gas, & & \\ T_n) = T_n \ldots) & \vdash & Boyle(Gas) = Pres(Gas, T_n) \times \\ & & Vol(Gas, T_n)\ (= K_n) \\ O_t & \vdash & Boyle(gas) ::= \lambda t.\ Pres(gas, t) \times \\ & & Vol(gas, t) = K(gas) \\ \exists i \neq j.\, O_t & \vdash & Boyle(Gas) - K_i \neq \\ & & Boyle(Gas) - K_j \end{array}$$

These formulae instantiate the trigger formulae with the following substitution:

$$\{Boyle/stuff,\ \langle Gas\rangle/\vec{s_i},\ \langle Gas\rangle/\vec{s_j},\ \langle gas\rangle/\vec{x},\ K/c,$$
$$Temp/V,\ \langle T_i\rangle/\vec{b_i},\ K_1/c_1,\ K_n/c_n\}$$

The variad is related to the inconstancy by means of having both the variad and the constancy as arguments of a new function F, i.e.

$$\nu(Boyle) ::= \lambda t, gas.\ F(K(gas), Temp(gas, t))$$

In the repaired ontologies, the repaired trigger formulae are therefore:

$$\nu(O_s(Temp(Gas, T_1) = T_1 \ldots)) \vdash (\nu(Boyle)(T_1))(Gas) = Pres(Gas, T_1) \times Vol(Gas, T_1) \; (= K_1)$$

$$\vdots \qquad \vdots$$

$$\nu(O_s(Temp(Gas, T_n) = T_n \ldots)) \vdash (\nu(Boyle)(T_n))(Gas) = Pres(Gas, T_n) \times Vol(Gas, T_1) \; (= K_n)$$

$$\nu(O_t) \vdash \nu(Boyle) ::= \lambda t, gas.\ F(K(gas)\ Temp(gas, t))$$

which breaks the derivation of the detected contradiction, as required.

As a result of the repair, the new function depends on both the parameters of Boyle's law and the temperature of the gas. Such a dependency drives the function dependence of the repaired law closer to that required for the ideal gas law. If F is properly approximated by regression, it should be discovered to be equivalent to the multiplication function, because the product of the pressure and the volume of a gas is directly proportional to the temperature for a fixed number of gas molecules, which gives

$$\nu(Boyle) ::= \lambda t, gas.\ K(gas) \times Temp(gas, t)$$

as required for the ideal gas law.

Application to the 3-body Problem

Leonhard Euler initially thought the orbits of the primary planets could be calculated on the basis of the exact solution of the *2-body problem*. More specifically, it was thought that the astronomical tables of the primary planets could be constructed by merely considering their attractions to the Sun alone (Taton and Wilson 1989). Euler's assumption changed when he began studying the inequalities of Jupiter's and Saturn's orbits, which is known as *The Great Inequality*. Euler later acknowledged that setting up the problem as a 2-body problem was not sufficient to correctly calculate the orbit of Saturn; instead, it would require at least solving the *3-body problem*, because Saturn's orbit was perturbed when it approached Jupiter. Although it was clear that there was a *causal relationship* between Jupiter's position and Saturn's orbit, we can still apply Inconstancy to this example because it can be viewed as a generic repair for scenarios involving causal links, where the variad can be seen as a missing cause.

To trigger the Inconstancy repair plan, we can model Euler's initial theory that the solution of the 2-body problem was sufficient in the predictive theoretical ontology, O_t. The calculation of the 2-body problem requires three pairs of input parameters: initial masses of the two bodies, $Mass(obj_1, t)$ and $Mass(obj_2, t)$; initial positions of the bodies, $Posn(obj_1, t)$ and $Posn(obj_2, t)$; and, initial velocities of the bodies, $Vel(obj_1, t)$ and $Vel(obj_2, t)$. The predicted orbit of Saturn, $\lambda t.\ X(Saturn, t)$, can therefore be represented as:

$$MVP2O(\lambda p \in \{Sun, Saturn\}. \langle Mass(p,t), Vel(p,t), Posn(p,t)\rangle) = \lambda t.\ X(Saturn, t))$$

where $MVP2O$ computes the orbit based on the masses, velocities and positions of the given planets. In the 2-body example, we look at those attributes of the Sun and Saturn only.

Since the perturbation was apparent when Saturn approached Jupiter, we want to obtain a collection of sensory data describing Jupiter at different states, i.e., at different positions, velocities and with different masses. Here, we will handle one variad at a time.

$$O_s(Posn(Jupiter, T_1) = J_1 \ldots) \vdash MVP2O(\lambda p \in \{Sun, Saturn\}, t.\ \langle Mass(p, T_1), Vel(p, T_1), Posn(p, T_1)\rangle) = Posn(Saturn, T_1)\ (= P_1)$$

$$\vdots \qquad \vdots$$

$$O_s(Posn(Jupiter, T_n) = J_n \ldots) \vdash MVP2O(\lambda p \in \{Sun, Saturn\}, t.\ \langle Mass(p, T_1), Vel(p, T_n), Posn(p, T_n)\rangle) = Posn(Saturn, T_n)\ (= P_n)$$

$$O_t \vdash MVP2O(\lambda p \in \{Sun, Saturn\}, t.\ \langle Mass(p,t), Vel(p,t), Posn(p,t)\rangle) = \lambda t.\ X(Saturn, t)$$

$$\exists i \neq j.\ O_t \vdash MVP2O(\lambda p \in \{Sun, Saturn\}. \langle Mass(p, T_i), Vel(p, T_i), Posn(p, T_i)\rangle) - P_i \neq MVP2O(\lambda p \in \{Sun, Saturn\}. \langle Mass(p, T_j), Vel(p, T_j), Posn(p, T_j)\rangle) - P_j$$

Using (5), the Inconstancy repair plan can be instantiated with the following substitution:

$$\{MVP2O/stuff,\ \lambda p \in \{Sun, Saturn\}. \langle Mass(p, T_i), Vel(p, T_i), Posn(p, T_i)\rangle/\vec{s_i},\ \lambda p \in \{Sun, Saturn\}. \langle Mass(p, T_j), Vel(p, T_j), Posn(p, T_j)\rangle/\vec{s_j},\ \lambda p \in \{Sun, Saturn\}. \langle Mass(p, T), Vel(p, T), Posn(p, T)\rangle/\vec{x}, \lambda t.\ X(Saturn, t)/c, Posn/V, \langle Jupiter, T_i\rangle/\vec{b_i},\ P_1/c_1, P_n/c_n\}$$

To provide a new definition of $MVP2O$, F is introduced to express the relationship between the 2-body theory and the position of Jupiter, such that the repaired prediction is consistent with the observed orbit:

$$\nu(\mathit{MVP2O}) ::= \lambda t.\ F(X(\mathit{Saturn}, t), \mathit{Posn}(\mathit{Jupiter}, t))$$

By applying the transformation rules, the repaired ontologies are:

$$\begin{aligned}
\nu(O_s(\mathit{Posn}(\mathit{Jupiter}, & \\
T_1) = J_1 \ldots) &\vdash \nu(\mathit{MVP2O})(T_1) = \mathit{Posn}(\\
& \quad \mathit{Saturn}, T_1) \\
\vdots \quad & \quad \vdots \\
\nu(O_s(\mathit{Posn}(\mathit{Jupiter}, & \\
T_n) = J_n \ldots) &\vdash \nu(\mathit{MVP2O})(T_n) = \mathit{Posn}(\\
& \quad \mathit{Saturn}, T_n) \\
\nu(O_t) &\vdash \nu(\mathit{MVP2O}) ::= \lambda t.\ F(X(\\
& \quad \mathit{Saturn}, t), \mathit{Posn}(\mathit{Jupiter}, t))
\end{aligned}$$

The discrepancy is resolved because the repaired ontologies are now consistent when Saturn's orbit is observed to change as the position of Jupiter changes. However, it does not model the complete set of input parameters required to calculate the solution of the general 3-body problem. To this end, observations of unexpected orbital perturbation under different velocities and masses of Jupiter will also be required. Of course, it seems unpractical to observe mass variations of Jupiter because its mass is almost constant and variations are not apparent, unlike the mass of black holes. For demonstration purposes, we will assume that such observations are available. If such sensory data is provided, both the repairs triggered by variations in the velocity and in the mass of Jupiter can be incrementally applied *over* the described repair, triggered by a variation in the position.

After the three repairs, the new *MVP2O* would be:

$$\begin{aligned}
&\nu(\nu(\nu(\\
&\mathit{MVP2O}))) ::= \lambda t_m.\ F_m(\lambda t_v.\ F_v(\lambda t_p.\ F_p(X(\mathit{Saturn}, \\
&\quad t_p), \mathit{Posn}(\mathit{Jupiter}, t_p)), \mathit{Vel}(\mathit{Jupiter}, \\
&\quad t_v)), \mathit{Mass}(\mathit{Jupiter}, t_m)).
\end{aligned}$$

where F_p, F_v and F_m are three different functions approximated using different parameters - namely, the observed positions, velocities and masses (assuming mass variations are apparent and can be observed) of Jupiter respectively; t_p, t_v and t_m are the respective time moments corresponding to those observations. This new definition of *MVP2O*, $\nu(\nu(\nu(\mathit{MVP2O})))$, therefore depends on the position, velocity and mass of Jupiter, as required by the general formula for the 3-body problem.

Conclusion

Described is the Inconstancy ontology repair plan, designed for resolving contradictions stemmed from the predicted independence in the predictive theory and the observed dependence of a function. It has been successfully evaluated and applied to a small but diverse set of examples from the history of physics. We have demonstrated how the repairs performed by this repair plan transform inconsistent ontologies into new theories that closely match true physical formulations. As our work is still in its early stages, we will require further investigation into the history of physics to identify both additional repair plans and case studies. The revised trigger proposed for Inconstancy is intuitively more general than the original, but further study is essential to confidently determine its effects on applicability. So far, our repair plans are fairly *ad hoc*, due to the lack of a principled theory generalising the atomic repair operations. Our notion of minimality is in its infancy, but defining it as a conservative extension is a promising start.

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