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by

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Informatics Research Report EDI-INF-RR-0194

School of Informatics **January 2004**

http://www.informatics.ed.ac.uk/

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This paper investigates the computational complexity of reasoning with English sentences featuring temporal prepositions, temporal subordinating conjunctions and the order-denoting adjectives 'first' and 'last'. A fragment of English featuring these constructions, called TPE, is defined by means of a context-free grammar. The phrase-structures which this grammar assigns to the sentences it recognizes can be viewed as formulas of an interval temporal logic, called TPL, and given intuitively correct semantics. It is shown that the satisfiability problem for TPL is NEXPTIME-complete.

Keywords : Natural language semantics, interval temporal logic, computational complexity

Temporal Prepositions and their Logic

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Abstract

This paper investigates the computational complexity of reasoning with English sentences featuring temporal prepositions, temporal subordinating conjunctions and the order-denoting adjectives first and last. A fragment of English featuring these constructions, called $\mathcal{TPE}$, is defined by means of a context-free grammar. The phrase-structures which this grammar assigns to the sentences it recognizes can be viewed as formulas of an interval temporal logic, called $\mathcal{TPL}$, and given intuitively correct semantics. It is shown that the satisfiability problem for $\mathcal{TPL}$ is NEXPTIME-complete.

1 Introduction

Consider the following sentences:

(1) An interrupt was received during every cycle

(2) The main process ran after the last cycle

(3) While the main process ran, an interrupt was received before loop 1 was executed for the first time.

These sentences speak of events and their temporal locations: of what happened and when. The principal devices they employ to encode this information are temporal prepositions, temporal subordinating conjunctions and the adjectives first and last. The aim of this paper is to answer the question: What is the computational complexity of reasoning with sentences encoding temporal information using such devices?

This question is of theoretical interest, because the events mentioned in (1)–(3)—cycles, executions of processes, receipts of interrupts—are *extended* in time;

*This paper was written during a visit by the author to the Institute for Communicating and Collaborative Systems, Division of Informatics, University of Edinburgh. The hospitality of the ICCS and the support of the EPSRC (grant reference GR/S22509) are gratefully acknowledged. The author would also like to thank Mark Steedman and David Brée for helpful discussions.

and temporal logics which deal with extended events—so-called *interval temporal logics*—typically exhibit high computational complexity. Thus, the best-known interval temporal logic, $\mathcal{HS}$ (Halpern and Shoham [6], see also Venema [19]), is undecidable, with little known about its decidable fragments. (For a forthcoming discussion, see [5]). Indeed, the best-known *decidable* interval temporal logic, ITL (Moszkowski [10]), assumes that its non-logical primitives are in fact point-events, and yet still has a non-elementary satisfiability problem. Given that the syntax of these logics has little affinity with that of temporal expressions in English, it is natural to ask whether the meanings of sentences such as (1)–(3) can be captured in a computationally manageable logic.

Further theoretical motivation comes from the side of natural language semantics. The formal semantics of temporal constructions in English has been addressed by a succession of researchers (Crouch and Pullman [2], Dowty [4], Hwang and Schubert [8], Kamp and Reyle [9], Ogihara [11], Stump [16], ter Meulen [17] to name but a few). Yet natural language semanticists typically employ whatever formalism is sufficiently expressive to capture the sentence-meanings they identify and sufficiently familiar to command the assent of the relevant academic community. In particular, most accounts of the semantics of temporal constructions in English represent sentence-meanings in a first-order language having variables which range over time-intervals and predicates corresponding to event-types and temporal order-relations; and such a logic is easily shown to be undecidable. Given the recent surge of interest in logical fragments of limited computational complexity, this situation is unsatisfactory. There are evident practical and theoretical reasons for developing the semantics of various natural language constructions, where possible, using formal systems of limited expressive power.

The plan of this paper is as follows. Section 2 presents an outline of the semantics of the English temporal constructions considered in this paper. Section 3 then uses a simple context-free grammar to define a fragment of English featuring these constructions; we call this fragment $\mathcal{TPE}$, a rough acronym for *temporal preposition English*. We show how the phrase-structures assigned to $\mathcal{TPE}$-sentences by this grammar can in fact be viewed as expressions in an interval temporal logic, which we call $\mathcal{TPL}$. Section 4 presents formal semantics for $\mathcal{TPL}$. Sections 5 and 6 provide matching upper and lower complexity-bounds for $\mathcal{TPL}$-satisfiability, showing that this problem is NEXPTIME-complete.

2 Semantics

In this section, we sketch an outline of the semantics of temporal prepositions and temporal subordinating conjunctions in English. We begin with the simplest cases.

2.1 Cascading and context

Consider the following sentences:

(4) An interrupt was received

(5) An interrupt was received during every cycle

(6) An interrupt was received during every cycle until the main process ran

(7) After the initialization phase, an interrupt was received during every cycle until the main process ran.

Sentence (4) is true just in case, at some time within some contextually given interval of interest, an interrupt was received. Interpreting the unary predicate int-rec so that it is satisfied by all and only those time intervals over which an interrupt was received, we may represent these truth-conditions by:

(8) $\exists J_0(\text{int-rec}(J_0) \wedge J_0 \subset I)$.

Throughout this paper, the letters I, J, ..., with or without decorations, range over time intervals, which we take to be closed, bounded, (non-empty) convex subsets of the real line.

The fragment of temporal English considered here deals only with events, as opposed to states—that is, only with telic as opposed to atelic eventualities (Vendler [18]; see, e.g. Steedman [15] for an extended discussion). The thesis that simple, event-reporting sentences are implicitly existentially quantified was proposed by Davidson [3], and is defended in Parsons [12]. These authors take the quantification in question to be over events rather than time intervals; but this issue may be ignored for present purposes. (A recent collection of papers on this topic can be found in Higginbotham *et al.* [7].) One could doubtless quibble about whether the $\subset$ in (8) should be $\subseteq$; however, the operative concepts seem too vague for this issue to admit of resolution.

Notice that the contextually given interval to which the quantification in (4) is limited is represented by the free variable I in (8). That is: a sentence meaning is a *temporal abstract*, which receives a truth-value (in a model) only relative to an interval of evaluation. It turns out that viewing sentence meanings in this way clarifies the logical relationships between the sentences (4)–(7). The following notation will help keep things concise. If I and J denote the intervals $[a, b]$ and $[c, d]$, respectively, with $a, b, c, d \in \mathbb{R} \cup \{-\infty, \infty\}$ and $a \leq c \leq d \leq b$, we let the terms $\text{init}(J, I)$ and $\text{fin}(J, I)$ denote the intervals $[a, c]$ and $[d, b]$, respectively. In other words, whenever $J \subseteq I$ is true, we take $\text{init}(J, I)$ to denote the initial segment of I up to the start of J, and $\text{fin}(J, I)$ to denote the final segment of I from the end of J. (Recall that intervals may be punctual.) Helping ourselves to a suitable signature of unary predicates of intervals, we may then formalize sentences (5)–(7) as follows:

(9) $\forall J_1(\text{cyc}(J_1) \wedge J_1 \subset I \rightarrow \exists J_0(\text{int-rec}(J_0) \wedge J_0 \subset J_1))$

(10)
$$\begin{array}{l}\iota J_2(\text{main}(J_2) \wedge J_2 \subset I, \\ \quad \forall J_1(\text{cyc}(J_1) \wedge J_1 \subset \text{init}(J_2, I) \rightarrow \exists J_0(\text{int-rec}(J_0) \wedge J_0 \subset J_1)))\end{array}$$

(11)
$$\begin{array}{l}\iota J_3(\text{init-phase}(J_3) \wedge J_3 \subset I, \\ \quad \iota J_2(\text{main}(J_2) \wedge J_2 \subset \text{fin}(J_3, I), \\ \qquad \forall J_1(\text{cyc}(J_1) \wedge J_1 \subset \text{init}(J_2, \text{fin}(J_3, I)) \rightarrow \\ \qquad\quad \exists J_0(\text{int-rec}(J_0) \wedge J_0 \subset J_1)))).\end{array}$$

The ι operator is the standard Russellian definite quantifier. We pass over the usual issues as to the faithfulness of this interpretation of definite quantification (either expressed or implied) in these sentences. Notice how the quantifiers introduced by successive temporal preposition phrases bind the temporal context variables associated with the sentence they modify. This cascading of restrictions on quantification, typical of iterated temporal preposition phrases, was pointed out in Pratt and Francez [13], and is discussed further in von Stechow [20].

2.2 Complications

It is impossible, within the space of a few pages, to do full justice to the complexities of the English temporal constructions featured in this paper. Nevertheless, some elaboration of the foregoing account is required; we confine ourselves to those features of greatest relevance to the ensuing computational analysis. For a more thorough guide to the linguistic subtleties surrounding temporal constructions in English, see e.g. Bennett [1] or Quirk *et al.* [14].

We begin with some remarks on the temporal preposition (or subordinating conjunction) before. The sentence

(12) An interrupt was received before the main process ran

is true in a temporal context I when there is a unique running of the main process during I, and an interrupt is received over some subinterval of I prior thereto. Ordinary usage is vague as to whether it is the start- or end-times of the events in question that are being compared. To resolve any uncertainly, we simply take (12) to require that some interrupt-event *finished* before the run of the main process *began.* We therefore propose to render the meaning of (12) by

(13) $$\iota J_1(\text{main}(J_1) \wedge J_1 \subset I,\ \exists J_0(\text{int-rec}(J_0) \wedge J_0 \subset \text{init}(J_1, I))).$$

Notice that these truth-conditions impose no limit on *how* long before the running of the main process the interrupt was received (except that imposed by the temporal context I). That is: before is here used in the sense of *some time before.* In some situations, however, before is more naturally taken to mean *just before* or *shortly before.* This latter sense reflects the possibility of adding a time-measure as a specifier, as in the phrase five minutes before. In this paper, we ignore this latter sense of before entirely: incorporating it into our account would involve us in a discussion of either vagueness or the semantics of temporal measure-phrases, both of which we choose to avoid.

Actually, the previous paragraph is misleading in glossing the sense of before assumed here as *some time before.* For the existential quantification in the meaning (13) of (12) is not provided by the before-phrase at all, but rather by the sentence An interrupt was received occurring in its scope; the before-phrase serves merely to specify a temporal context to which that quantification is restricted. In fact, there is no reason why this quantification need be existential at all, thus:

(14) An interrupt was received during every cycle before the main process ran.

We take (14) to have the meaning (10); that is, we take it to be synomymous with (6). Here again, the before-phrase in (14) serves merely to identify a temporal context to which the quantification in its scope is restricted; in particular, it provides no universal quantification of its own.

As for before, so for until: until-phrases serve only to create temporal contexts restricting the quantification provided by the sentences in their scope; but they do not provide that quantification. This is most aparent by considering the pair of sentences (5) and (6), where the universal quantification evidently arises from the determiner every. This treatment of until may surprise readers familiar with so-called until-operators in temporal logic, whose semantics do typically contribute universal quantification. Apparently, there is an association of until with universal quantification, at least in the minds of temporal logicians; and it is natural to ask how this apparent association can be reconciled with the view adopted here.

The answer is as follows. Sentence (5), which the until-phrase in sentence (6) modifies, is *downward monotonic*: if it is true over some interval I, then it is also true over all subintervals of I. (Downward monotonicity is, of course, characteristic of sentences which universally quantify over subintervals.) It transpires that until-phrases *require* a downward-monotonic scope, as witnessed by the anomalous:

(15) ? An interrupt was received until the main process ran.

(16) ? An interrupt was received during some cycle until the main process ran.

Thus, on our account, the universal quantification—or more accurately, downward monotonicity—is not provided by until; but the presence of until requires it to be provided by something else. Before, of course imposes no such requirement, as we have seen. Thus, on our account, the difference between before (in the sense adopted here) and until, lies not in their contribution to truth-conditions, but merely in the situations in which they can be used. Actually, the linguistic data on until are rather awkward, and appear to fit no very appealing logical pattern. In particular, downward monotonicity is not always sufficient for applicability of until-phrases (see e.g. Zucchi and White [21]). The exploration of this issue—and indeed of the myriad other differences between before and until—lies outside the scope of the present enquiry.

The subordinating conjunction when creates another sort of difficulty. When serves primarily to indicate proximity between the the events identified in its scope and complement, thus:

(17) An interrupt was received when the main process ran.

Sentences such as (17) in fact impose remarkably loose constraints on the temporal relation between the events in question, as various writers have noted. But whatever the final verdict on the nature of those constraints, we cannot usefully treat the associated vagueness in the present paper, and some further regimentation is necessary. To simplify issues, we treat (17) as synonymous with

(18) An interrupt was received *while* the main process ran.

and give it the semantics

(19) $\iota J_1(\text{main}(J_1) \wedge J_1 \subset I,$
$\exists J_0(\text{int-rec}(J_0) \wedge J_0 \subset J_1)).$

Our excuse for doing so is simply that containment is an easier relation to work with than approximate collocation. Readers who find this expedient too brutal can simply omit when from our fragment.

We have already discussed quantification in the *scope* of temporal prepositions and subordinating conjunctions; we now move to the issue of quantification in their *complements*. Temporal prepositions have noun-phrase complements which typically include determiners; and these determiners contribute quantification to the meanings of sentences containing them. This is evident, for example, with the occurrences of during *every* cycle in (5)–(7), which contribute the universal quantifiers to (9)–(11). Temporal subordinating conjunctions, by contrast, take sentential complements lacking any overt analogue of a determiner; and the question therefore arises as to how the variables in these complements get quantified.

The answer is that the complements of temporal subordinating conjunctions are (almost always) taken to be *definitely* quantified—i.e. bound by an ι-operator. Thus, until the main process ran in (6) is interpreted as *until the unique time over which the main process ran*, as reflected by the ι-quantifier in (10). It may seem harsh to count (6) as *false* if there are two runs of the main process within the temporal context; it would perhaps be fairer to interpret the relevant until-phrase as picking out the period before the *first* time over which the main process ran. But since this facility is available in our fragment anyway, as discussed in Section 2.3, the issue need not detain us.

The obvious exception to the definite quantification of complements of temporal subordinating conjunctions is whenever. Thus, we take

(20) Whenever the main process ran, an interrupt was received

to have the truth-conditions

(21) $\forall J_1(\text{main}(J_1) \wedge J_1 \subset I \rightarrow \exists J_0(\text{int-rec}(J_0) \wedge J_0 \subset J_1))).$

That is: the variable contributed by the complement of the whenever-phrase is universally quantified. In the sequel, we shall assume that all quantification of the complements of temporal subordinating conjunctions is definite, except in the case of whenever, where is it universal. Note that we are mimicking our earlier discussion of when in again taking the operative temporal relation here to be containment rather than approximate collocation. As before, this represents a certain deviation from ordinary usage; again, however, we cannot sensibly deal with vague truth-conditions here, and so we pass over the issue. Interestingly enough, the English word whilever does not exist.

Some temporal prepositions have been conspicuous by their absence from the foregoing discussion. The temporal prepositions on and in, in phrases such as on Mondays or in January, are specific to certain categories of arguments, but are otherwise equivalent to during: these may be ignored for the purposes of this paper. The preposition at, which in English is used in conjunction with clock-times (and some religious festivals) may also fall into this category, though there are further complications here concerning its inherent approximateness. The propositions for and in, in phrases such as for/in five minutes, take as complements temporal measure-phrases. These lie outside the scope of the logic considered here.

The preposition by, in its temporal sense, functions analogously to until, except that it prefers upward-monotonic sentences in its scope; moreover, like until, it dislikes complements which are not explicitly temporal, thus:

(22) An interrupt was received by 5 o'clock

(23) ? An interrupt was received by the first cycle.

(Note that (23) has a perfectly natural reading in which by is interpreted non-temporally.) In addition, by exhibits interesting interactions with aspect:

(24) The main process ran/had run/was running by 5 o'clock.

Finally, we observe that by occurs frequently in the construction by the time ... with a sentential complement, with the same preference for qualifying upward-monotonic sentences. Dealing with the rather difficult behaviour of by in our fragment would complicate the grammar without adding anything of logical interest, and so we ignore it.

In some respects, the mirror-image of both until and by is since:

(25) An interrupt has been received since the main process ran

(26) An interrupt has been received during every cycle since the main process ran.

(When used in its temporal sense, since requires the sentence in its scope to have perfect aspect.) Unlike until and by, however, since resists embedding in contexts established by quantification, as we see by comparing

(27) During every cycle, an interrupt did not occur until the main process ran

(28) ? During every cycle, an interrupt has/had not occurred since the main process ran.

Because of these complications, we do not include since in our fragment. However, we do include after, which we take (again, ignoring some linguistic subtleties) to function as a mirror image of before. Given the inclusion of after, our omission of since does not affect the fragment's expressive power.

2.3 First and Last

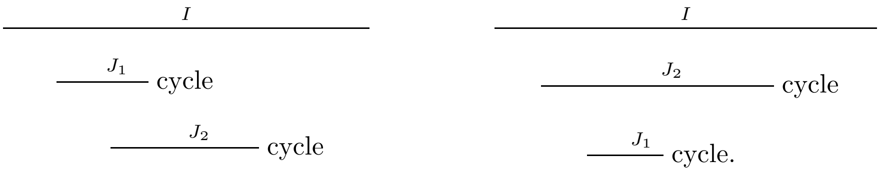
Our fragment will also contain sentences such as

(29) An interrupt was received during the *first* cycle

(30) An interrupt was received before the main process ran for the *last* time.

We briefly consider the issue of assigning truth-conditions to such sentences.

Suppose that, in the relevant temporal context I, there is an unambiguously first cycle: that is, a cycle which begins and ends before all the others. Then (29) asserts that, if J is the interval over which this cycle occurs, then an interrupt was received over some sub-interval of J. A corresponding account can of course be given for (30). Problems arise, however, when there is no unambiguously first cycle within I. Suppose, for example, cycles occur during intervals J_1, J_2, and nowhere else, in either of the following arrangements. (In such diagrams, left-to-right arrangement depicts temporal order; vertical arrangement has no significance.)



It is unclear what the truth-value of (29) should be in such cases. Apparently, we need to legislate.

We take the mathematically simplest way out. Since we may assume that only *finitely* many events of any given type e occur within a given interval I, we proceed as follows. Let $\mathcal{J}$ be the collection of all subintervals of I over which an event of type e occurs, and assume $\mathcal{J}$ is nonempty. Since $\mathcal{J}$ is by hypothesis finite, we can select the (non-empty) subset $\mathcal{J}'$ whose elements have the (unique) earliest end-point. In case $\mathcal{J}'$ has more than one element, let us select the unique element $J \in \mathcal{J}'$ whose start-point is latest. Thus, J is the *smallest* of the *earliest-ending* sub-intervals I of type e. In the sequel, then, we interpret the phrase the first e, within a temporal context I, to pick out this interval. (In the situations depicted above, these are the intervals marked J_1.) Similarly, we interpret the phrase the last e, within a temporal context I containing at least one occurrence of e, to pick out the *smallest* of the *latest-beginning* sub-intervals of I over which an e-event occurs. To re-iterate, we are simply legislating here in the most convenient way in cases where native-speaker intuition returns an unclear verdict.

3 A Fragment of Temporal English

The task of this section is to define a fragment of temporal English. We do this by writing a definite clause grammar to recognize its sentences. This grammar assigns phrase-structures to these sentences in the familiar way, and we shall see that, following some cosmetic re-arrangement, these phrase-structures can be regarded as expressions in a formal language. This formal language will constitute the basis of the temporal logic $\mathcal{TPL}$ defined in Section 4.

3.1 Delineating the fragment

We begin with the simplest types of sentences in our fragment:

(31) An interrupt was received

(32) An interrupt was not received.

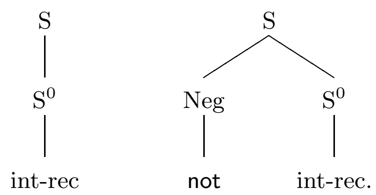
For present purposes, sentence (31) is taken as atomic: that is, we ignore its internal structure. Accordingly we treat such sentences as vocabulary items, of class S^0, and write the grammar rules:

$$S \rightarrow S^0 \qquad\qquad S^0 \rightarrow \text{an interrupt was received/int-rec.}$$

Furthermore, the only property of sentence (32) which concerns us is its relation to (31): that is, we ignore other aspects of its structure. Accordingly, we pretend that (32) is obtained by simply *prefixing* the word not to (31), and write the grammar rules

$$S \rightarrow \text{Neg}, S^0 \qquad\qquad \text{Neg} \rightarrow \text{not.}$$

This expedient removes needless clutter from our grammar, while affecting nothing of logical substance. (It is a simple exercise to restore the clutter.) Thus, our grammar assigns (31) and (32) the respective phrase-structures:



These phrase-structure diagrams feature the symbol int-rec, as specified in the above lexical entry for an interrupt was received. This symbol may be regarded as an abbreviation.

Temporal prepositions belong in our grammar to the category P_N, and occur in phrases such as

(33) during every cycle

(34) after the initialization phase

(35) before the first interrupt.

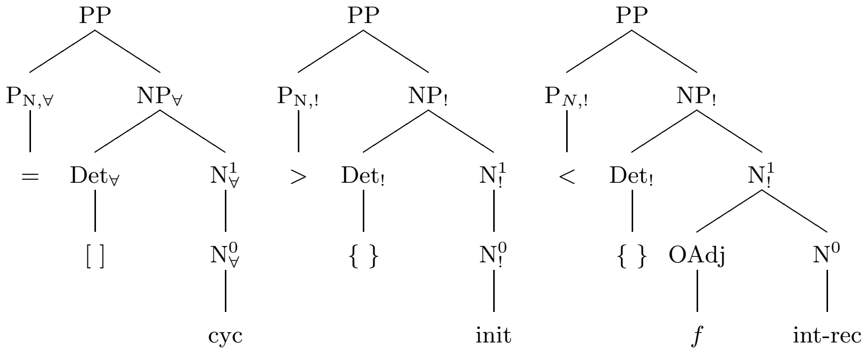
Nominal expressions such as cycle, initialization phase and interrupt are taken to be of (lexical) category N^0 and to denote event-types in the same way as items of category S^0. Again, we regard them as structureless:

$$N^0 \rightarrow \text{cycle/cyc} \quad N^0 \rightarrow \text{initialization phase/init} \quad N^0 \rightarrow \text{interrupt/int-rec.}$$

We allow these expressions to be optionally modified (once) by the order-specifying adjectives first and last, resulting in a phrase which in turn combines with a determiner to produce the complement of a temporal preposition. Accordingly, we write the grammar rules:

$\mathrm{NP}_D \to \mathrm{Det}_D, \mathrm{N}^1_D$	$\mathrm{PP} \to \mathrm{P}_{\mathrm{N},D}, \mathrm{NP}_D$	$\mathrm{Det}_\forall \to$ every/[ ]	$\mathrm{P}_{\mathrm{N,D}} \to$ during/=
$\mathrm{N}^1_D \to \mathrm{N}^0$	OAdj → first/f	$\mathrm{Det}_! \to$ the/{ }	$\mathrm{P}_{\mathrm{N},!} \to$ after/>
$\mathrm{N}^1_! \to \mathrm{OAdj}, \mathrm{N}^0$	OAdj → last/l	$\mathrm{Det}_\exists \to$ some/⟨ ⟩	$\mathrm{P}_{\mathrm{N},!} \to$ before/<.

Thus, our grammar assigns (33)–(35) the respective phrase-structures:



As before, we have replaced terminal nodes with the corresponding mnemonics to the right of the slashes in the lexicon; again, this removes clutter from the diagrams.

The variable subscript D in the above rules ranges over the set $\{\forall, \exists, !\}$. These items are simply tags indicating subcategorization of NP, N^1 and P_N. This subcategorization restricts the use of determiners in two ways. First, it requires that phrases involving first and last only ever combine with the definite article. This requirement reflects the observation that (outside university mathematics departments) locutions such as during a first interrupt and during every first interrupt are anomalous.

Our second restriction on the use of determiners requires that complements of the temporal prepositions until, before and after also incorporate the definite article. For until, this requirement serves to rule out some clearly anomalous sentences (it is the italicized *every* which causes the problem):

(36) ? An interrupt occurred during every cycle until *every* reset point.

For before and after, the requirement reflects our earlier decision to interpret before in the sense of *some time before*, rather than *shortly before*. For common usage (again: professional mathematicians excepted) does not take the sentences

(37) An interrupt was received before every reset point

(38) An interrupt was received before the first reset point

to be equivalent in contexts where there is a unique first reset point, as our assumed sense of before would require. We conclude that the term before can only have the *shortly-before* sense in (37), and so we banish that sentence from our fragment. Admittedly, existentially quantified complements with these prepositions sound fine, even with our chosen sense of before:

(39) An interrupt occurred before *some* reset point.

(40) An interrupt occurred during every cycle until *some* reset point

Indeed, such sentences could be admitted into our fragment without compromising the complexity-theoretic results derived below. However, banning sentences such as (36) while admitting those such as (40) will generate a logical fragment not fully closed under negation; and, while such fragments are unproblematic in principle, they tend to make for notational and conceptual clutter. For simplicity, therefore, we duck the issue, and simply decree that these temporal prepositions require complements with the definite article.

Temporal subordinating conjunctions belong in our grammar to the category $\mathrm{P_S}$, and occur in phrases such as

(41) before the main process ran

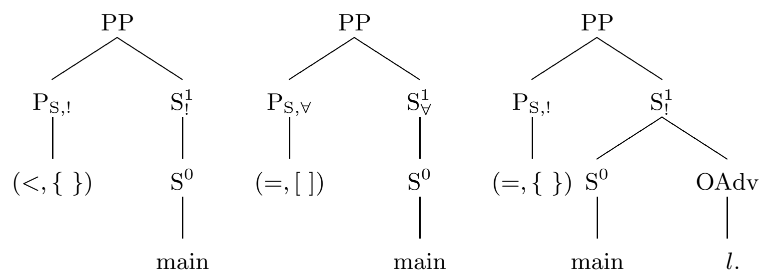
(42) whenever the main process ran

(43) while the main process ran for the last time.

Unmodified sentential complements are taken to be atomic, again of category S_0. Our grammar permits modification (once) of these complements by the adverbials for the first/last time, analogous to the modification of nominal complements by the adjectives first/last. Negation is not allowed in $\mathrm{P_S}$-complements. This restriction reflects the fact that such complements are generally interpreted as providing a definitely quantified event-type (universally quantified in the case of whenever); and negated event-types make little sense in this regard. Accordingly, we write the grammar rules:

$\mathrm{PP} \to \mathrm{P}_{\mathrm{S},D},\ \mathrm{S}^1_D$	$\mathrm{OAdv} \to$ for the first time$/f$	$\mathrm{P}_{\mathrm{S},!} \to$ when$/(=, \{\ \})$
$\mathrm{S}^1_! \to \mathrm{S}^0,\ \mathrm{OAdv}$	$\mathrm{OAdv} \to$ for the last time$/l$	$\mathrm{P}_{\mathrm{S},!} \to$ before$/(<, \{\ \})$
$\mathrm{S}^1_D \to \mathrm{S}^0$		$\mathrm{P}_{\mathrm{S},\forall} \to$ whenever$/(=, [\ ])$,

thus assigning (41)–(43) the respective phrase-structures:



Recall that, alone among temporal subordinating conjunctions, whenever is associated with universal, rather than definite, quantification of its complement. That is why the grammar rule for whenever incorporates the symbol [ ], rather than { }, to the right of the slash. The motivation for these mnemonics will become clear in Section 4.

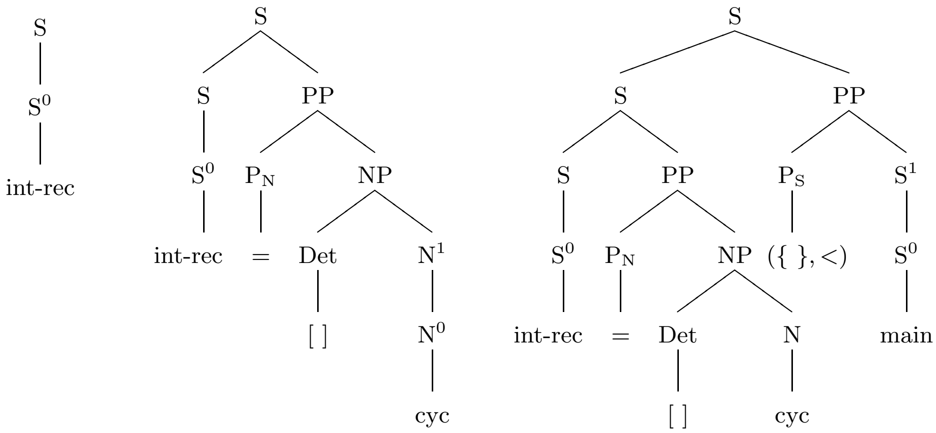


Figure 1: The structure of sentences (4)–(6)

Apart from the absence of determiners in subordinate clauses, temporal prepositions and temporal subordinating conjunctions are thus given parallel treatments. We allow that expressions of categories S^0 and N^0 may correspond to the *same* event-type, as indicated by the mnemonics in the lexicon, thus:

$$S^0 \rightarrow \textsf{the main process ran}/\text{main}$$
$$N^0 \rightarrow \textsf{run of the main process}/\text{main}.$$

Since we want to finesse issues of subsentential and subnominal structure, we leave it to grammar-writers' common sense to spot such nominalizations where they occur. The task of providing a more complex grammar to automate this job is independent of the issues addressed here.

Finally, we require grammar rules for applying prepositions and subordinating conjunctions to the sentences in their scope. In addition, we allow coordination of sentences (not lower phrases) using and and or. There are no surprises here:

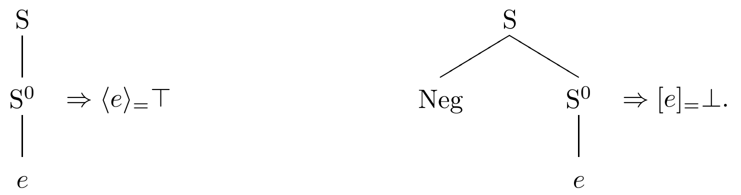
$$S \rightarrow S, PP \quad S \rightarrow S, Conj, S \quad Conj \rightarrow \textsf{and}/\wedge \quad Conj \rightarrow \textsf{or}/\vee.$$

Fig 1 shows the phrase-structures of sentences (4)–(6). (We have suppressed subcategorization information for clarity.) As usual, the leaf nodes have been replaced by the symbolic abbreviations specified in the lexicon. We note in passing that we have ignored the phenomenon of *preposed* preposition-phrases, as in Sentence (7). It should be obvious that this defect can be easily rectified.

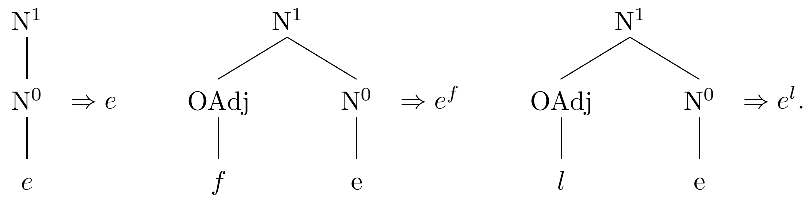
This completes our explanation of the fragment of English studied in this paper. We dub this fragment $\mathcal{TPE}$, a rough acronym for *temporal preposition English*; the full list of grammar rules is given in the Appendix to this paper. Technically, of course, $\mathcal{TPE}$ is not a fragment, but a *family* of fragments: one of each choice of lexicon for the terminal categories N^0 and S^0. In practice, however, we may ignore this technicality.

3.2 Interpreting the fragment

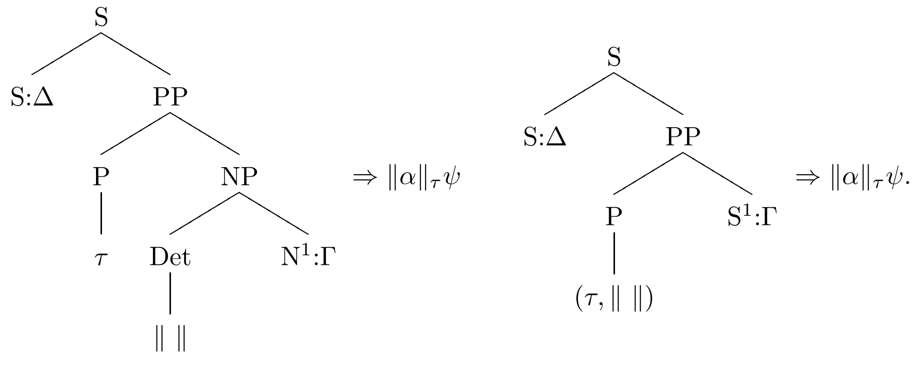
Our strategy is to treat phrase-structures in $\mathcal{TPE}$ as logical forms—that is, as formulas in a language for which a recursive semantics can be given in the style due to Tarski. To this end, we subject $\mathcal{TPE}$ phrase-structures to some minor geometrical re-arrangement. Any phrase structure whose root is labelled S and which dominates no PP will be re-written more compactly as one of the forms $\langle e\rangle_= \top$ or $[e]_= \bot$ as follows:



Similarly, any phrase-structure whose root is labelled N^1 will be re-written more compactly as one of the forms e, e^f or e^l as follows:



(Phrase-structures whose root is labelled S^1 will be re-written analogously.) Phrase-structures immediately dominating PP-phrases will be re-written more compactly in the form $\|\alpha\|_\tau\psi$ as follows, where ψ is the result of re-witing the structure Δ, α the result of re-witing the structure Γ, τ is one of the symbols $<$, $>$, or $=$, and $\|\ \|$ is one of the bracket-pairs $\langle\ \rangle$, $[\ ]$ or $\{\ \}$:



Finally, sentences involving and and or will be re-written more compactly as expressions with major connectives $\wedge$ and $\vee$ in the obvious way.

For example, subjecting the three trees drawn in Fig. 1 to this re-arrangement we obtain the respective formulas:

$$\langle\text{int-rec}\rangle_{=}\top \qquad [\text{cyc}]_{=}\langle\text{int-rec}\rangle_{=}\top \qquad \{\text{main}\}_{<}[\text{cyc}]_{=}\langle\text{int-rec}\rangle_{=}\top.$$

Apart from some unusual brackets and decorations, which will be explained later, the results of this re-arrangement look remarkably like formulas of propositional dynamic logic, with the event-classifying mnemonics occupying the place of atomic programs. So they look; and so they are. We shall give a standard account of the semantics of these formulas along the lines of the usual semantics for propositional dynamic logic. We stress (though it is obvious) that no information has been created or destroyed in this re-arrangement process: it is a simple graphical matter of replacing an unfamiliar logical typography with a more familiar (and more compact) one. We could have stuck with trees if we had really wanted.

Let us take stock. In Section 2, we proposed truth-conditions for a range of sentences involving temporal prepositions, temporal subordinating conjunctions, and the order-denoting adjectives first and last. In this section, we have formalized the English fragment we are working with using a simple context-free grammar. We observed that the phrase-structures which this grammar associates with the sentences it recognizes can be re-arranged as formulas of a language resembling propositional dynamic logic. Of course, the point of this re-arrangement is that the resulting formulas can be given a formal semantics which reproduces the truth-conditions proposed in Section 2. It is to this task we now turn.

4 The Temporal Logic

The previous section explained how PPs in $\mathcal{TPE}$ can be regarded as modal operators of the form $\|\alpha\|_\tau$, where α is an expression of one of the forms e, e^f or e^l, $\|\ \|$ is one of $\langle\ \rangle$, $[\ ]$ or $\{\ \}$, and τ is one of $=$, $<$ or $>$. However, we have already agreed to restrictions on the quantification in PP-complements which ensure that, if $\tau \in \{<,>\}$ or if α has one of the forms e^f, e^l, then $\|\ \|$ is $\{\ \}$. In addition, to avoid clutter, we drop the $=$-subscripts, e.g. writing $[e]$ instead of $[e]_=$. This cuts down the set of modal operators to the forms

$$\langle e\rangle, [e], \{e\}, \{e\}_\tau, \{e^\omega\}, \{e^\omega\}_\tau,$$

where $\tau \in \{<,>\}$ and $\omega \in \{f,l\}$.

In the sequel, let E be a fixed infinite set. We refer to elements of E as *event-atoms*.

Definition 1. Let e range over the set E of event-atoms. We define the categories of *event-relation* α and *formula* ϕ by the syntax:

$$\alpha := e \mid e^f \mid e^l;$$
$$\phi := \langle e\rangle\phi \mid [e]\phi \mid \{\alpha\}\phi \mid \{\alpha\}_>\phi \mid \{\alpha\}_<\phi \mid \phi\wedge\phi' \mid \phi\vee\phi' \mid \neg\phi \mid \top \mid \bot.$$

We take the language $\mathcal{TPL}$ to be the set of formulas, so defined.

This syntax corresponds exactly to that of the fragment of English $\mathcal{TPE}$, except in one detail, namely, the inclusion of Boolean negation. The availability of negation tidies up the logical analysis by ensuring the usual duality of the satisfiability and entailment problems. In the sequel, we avail ourselves of the Boolean connectives $\rightarrow$ and $\leftrightarrow$, understood as abbreviations in the usual way. This aids readability only: in fact, a simple check shows that $\neg$ is not required for the lower-complexity bound obtained below, so that our conclusions about the complexity of reasoning in $\mathcal{TPE}$ are not compromised.

Recall that $\mathcal{I}$ denotes the set of *intervals*, that is, the set of closed, bounded, convex (non-empty) subsets of $\mathbb{R}$. We continue to use the (partial) functions $\text{init}(J, I)$ and $\text{fin}(J, I)$ as before.

Definition 2. A $\mathcal{TPL}$*-interpretation* (henceforth: *interpretation*) is a finite subset of $\mathcal{I} \times E$. For any $J \in \mathcal{I}$, we write $\mathcal{A}(J)$ for $\{e \in E \mid \langle J, e\rangle \in \mathcal{A}\}$, and for any $e \in E$, we write $\mathcal{A}(e)$ for $\{J \in \mathcal{I} \mid \langle J, e\rangle \in \mathcal{A}\}$.

The motivation for restricting attention to finite models is simply that we have in mind situations in which event-atoms denote everyday event-types instantiated in finite contexts. Interpretations in which infinitely many events of a given type occur in a finite space of time are of no interest.

We now turn to the interpretation of event-relations. Recalling our (rather artificial) stipulations about the meanings of words first and last applied to event-types of which there *is* no unambiguously first or last instance, we adopt the following terminology.

Definition 3. Let I be an interval and $J \subset I$ where J satisfies some property $\mathcal{P}$. We say that $J = [a, b]$ is the *minimal-first* subinterval of I satisfying $\mathcal{P}$ just in case for every $J' = [a', b'] \subset I$ satisfying $\mathcal{P}$, either $b < b'$ or $b = b'$ and $a \geq a'$. Likewise, we say that $J = [a, b]$ is the *minimal-last* subinterval of I satisfying $\mathcal{P}$ just in case for every $J' = [a', b'] \subset I$ satisfying $\mathcal{P}$, either $a > a'$ or $a = a'$ and $b \leq b'$.

Definition 4. Let α be an event-relation, $\mathcal{A}$ an interpretation, and $I, J \in \mathcal{I}$. We define $\mathcal{A} \models_{I,J} \alpha$ by cases as follows:

1. $\mathcal{A} \models_{I,J} e$ iff $J \subset I$ and $e \in \mathcal{A}(J)$
2. $\mathcal{A} \models_{I,J} e^f$ iff $\mathcal{A} \models_{I,J} e$ and J is the minimal-first such interval;
3. $\mathcal{A} \models_{I,J} e^l$ iff $\mathcal{A} \models_{I,J} e$ and J is the minimal-last such interval.

It is obvious that, since $\mathcal{A}$ is finite, if there exists any $J \subset I$ such that $\langle J, e\rangle \in \mathcal{A}$, then the minimal-first and minimal-last such J exist and are unique.

We are now ready to give the truth-conditions for formulas in $\mathcal{TPL}$.

Definition 5. Let ϕ be a formula, $\mathcal{A}$ an interpretation, and $I \in \mathcal{I}$. We define $\mathcal{A} \models_I \phi$ recursively as follows:

1. $\mathcal{A} \models_I \langle e\rangle\psi$ iff for some J, $\mathcal{A} \models_{I,J} e$ and $\mathcal{A} \models_J \psi$;

2. $\mathcal{A} \models_I [e]\psi$ iff for all J, $\mathcal{A} \models_{I,J} e$ implies $\mathcal{A} \models_J \psi$;
3. $\mathcal{A} \models_I \{\alpha\}\psi$ iff there is a unique $J \subset I$ such that $\mathcal{A} \models_{I,J} \alpha$, and for that J, $\mathcal{A} \models_J \psi$;
4. $\mathcal{A} \models_I \{\alpha\}_< \psi$ iff there is a unique $J \subset I$ such that $\mathcal{A} \models_{I,J} \alpha$, and for that J, $\mathcal{A} \models_{\mathrm{init}(J,I)} \psi$;
5. $\mathcal{A} \models_I \{\alpha\}_> \psi$ iff there is a unique $J \subset I$ such that $\mathcal{A} \models_{I,J} \alpha$, and for that J, $\mathcal{A} \models_{\mathrm{fin}(J,I)} \psi$;
6. The usual rules for $\top$, $\bot$, $\wedge$, $\vee$ and $\neg$.

If $\mathcal{A} \models_I \phi$, we say that ϕ is *true* at I in $\mathcal{A}$. If, for all $\mathcal{A}$ and I, $\mathcal{A} \models_I \phi$ implies $\mathcal{A} \models_I \phi'$ we say that ϕ *entails* ϕ'. If ϕ and ϕ' entail each other, we say they are *logically equivalent* and write $\phi \equiv \phi'$. If Φ is a set of formulas, we write $\mathcal{A} \models_I \Phi$ if $\mathcal{A} \models_I \phi$ for all $\phi \in \Phi$; Φ is said to be *satisfiable* if some such $\mathcal{A}$ and I exist.

This completes the formal specification of the logic $\mathcal{TPL}$. Remember that the phrase-structure of every sentence of the English temporal fragment $\mathcal{TPE}$ *is* a $\neg$-free $\mathcal{TPL}$-formula; conversely, every $\neg$-free $\mathcal{TPL}$-formula *is* the phrase-structure of a sentence of $\mathcal{TPE}$. It is transparent that, on the above semantics for $\mathcal{TPL}$, the phrase-structures (formulas) which the grammar of $\mathcal{TPE}$ assigns to sentences (4)–(7) are equivalent to the truth-conditions (8)–(11) proposed in Section 2.

We conclude this section with some simple logical equivalences in $\mathcal{TPL}$.

Lemma 1. *For all $e \in E$, $\phi \in \mathcal{TPL}$, $\tau \in \{<,>\}$, $\omega \in \{t,f\}$:*

$$\neg\langle e\rangle\phi \equiv [e]\neg\phi \qquad \neg[e]\phi \equiv \langle e\rangle\neg\phi$$
$$\neg\{e\}_\tau\phi \equiv \neg\{e\}\top \vee \{e\}_\tau\neg\phi \qquad \neg\{e^\omega\}_\tau\phi \equiv [e]\bot \vee \{e^\omega\}_\tau\neg\phi$$

Proof. Trivial. □

5 Upper Complexity Bound

The aim of this section is to show that the satisfiability problem for $\mathcal{TPL}$ is in NEXPTIME. This is achieved by establishing an exponential bound on the size of satisfying structures.

Lemma 2. *Every $\mathcal{TPL}$-formula is equivalent to one in which $\neg$ appears only in subformulas of the forms $\neg\{e\}\top$.*

Proof. The logical equivalences of Lemma 1, together with familiar propositional validities, allow negations to be moved successively inwards until the desired form is reached. □

Definition 6. Let $\mathcal{A} \neq \emptyset$ be a structure. The *depth* of $\mathcal{A}$ is the greatest m for which there exist $J_1 \supset \ldots \supset J_m$ with $\mathcal{A}(J_i) \neq \emptyset$ for all i $(1 \leq i \leq m)$. If $\mathcal{A}$ is empty, we take its depth to be 0.

Lemma 3. *Let ϕ be a formula, $\mathcal{A}$ a structure and I an interval, such that $\mathcal{A} \models_I \phi$. Then there exists a structure $\mathcal{A}^* \subseteq \mathcal{A}$ with depth at most $O(|\phi|^2)$ such that $\mathcal{A}^* \models_I \phi$.*

Proof. We may assume that ϕ has the form guaranteed by Lemma 2. Let Φ be the set of subformulas of ϕ. For every event-atom e and every interval J, define

$$L(J) = \{\psi \in \Phi \mid \mathcal{A} \models_J \psi\}$$
$$L^*_e(J) = L(J) \setminus \bigcup\{L(K) | K \subset J, K \in \mathcal{A}(e)\}.$$

Thus, $L^*_e(J)$ records which subformulas of ϕ are true at an interval J, ignoring those subformulas which are true at subintervals of J satisfying e. Say that a pair $\langle J, e\rangle \in \mathcal{A}$ is *redundant* if $L_e(J) = \emptyset$ and there exist $K, K' \in \mathcal{A}(e)$ such that $K \subset K' \subset J$. Now set

$$\mathcal{A}^* = \mathcal{A} \setminus \{\langle J, e\rangle \mid \langle J, e\rangle \text{ is not redundant}\}.$$

It is obvious that, if $J \subset J'$ with $J, J' \in \mathcal{A}(e)$, then $L_e(J)$ and $L_e(J')$ are disjoint. It follows that the depth of $\mathcal{A}^*$ is bounded by $m(m' + 2)$, where m is the number of event-atoms occurring in ϕ and m' the number of subformulas of ϕ. It thus suffices to show that, for all I and all $\psi \in \Phi$, $\mathcal{A} \models_I \psi$ implies $\mathcal{A}^* \models_I \psi$.

We proceed by induction on the complexity of ψ. The base cases are of the forms $\psi = \top, \bot, \neg\{e\}\top$. The first two of these are trivial. For the case $\psi = \neg\{e\}\top$, suppose $\mathcal{A} \models_I \psi$. If there is no $J \subset I$ with $J \in \mathcal{A}(e)$, then since $\mathcal{A}^* \subseteq \mathcal{A}$, we certainly have $\mathcal{A}^* \models_I \psi$. Otherwise, there exist $J \subset I$ and $J' \subset I$ with $J \neq J'$ and $J, J' \in \mathcal{A}(e)$. If either J or J' is redundant, there exist $K \subset K' \subset I$ with $K, K' \in \mathcal{A}^*(e)$; and if neither is redundant, $J, J' \in \mathcal{A}^*(e)$. Either way, $\mathcal{A}^* \models_I \psi$.

The recursive cases are of the forms $\psi = [e]\pi, \langle e\rangle\pi, \{\alpha\}_\tau\pi$, where α is of the forms e, e^f or e^l. For the case $\psi = [e]\pi$, we need only observe that $\mathcal{A}^* \subseteq \mathcal{A}$. For the case $\psi = \langle e\rangle\pi$, suppose $\mathcal{A} \models_I \psi$. Then then there exists $J \subset I$ such that $J \in \mathcal{A}(e)$ and $\mathcal{A} \models_J \pi$. By the finiteness of $\mathcal{A}$, choose such a J which is minimal under the order $\subset$, so that $J \in \mathcal{A}^*(e)$. By inductive hypothesis, $\mathcal{A}^* \models_J \pi$; hence $\mathcal{A}^* \models_I \psi$. For the case $\psi = \{e\}\pi$, suppose $\mathcal{A} \models_I \psi$. Then there exists a unique $J \subset I$ such that $J \in \mathcal{A}(e)$; and for this J, $\mathcal{A} \models_J \pi$. In particular, there is no $K \subset J$ such that $K \in \mathcal{A}(e)$, whence $J \in \mathcal{A}^*(e)$. By inductive hypothesis and the fact that $\mathcal{A}^* \subseteq \mathcal{A}$, we then easily have $\mathcal{A}^* \models_I \psi$. The remaining cases are dealt with exactly as for $\psi = \{e\}\pi$, noting, in particular, that $\mathcal{A} \models_{I,J} e^f$ implies $\mathcal{A}^* \models_{I,J} e^f$ and $\mathcal{A} \models_{I,J} e^l$ implies $\mathcal{A}^* \models_{I,J} e^l$. □

Theorem 1. *Let ϕ be a formula of $\mathcal{TPL}$. If ϕ is satisfiable, then ϕ is satisfied in a structure of size bounded by $2^{p(|\phi|)}$, for some fixed polynomial p.*

Proof. Suppose that $\mathcal{A} \models_{I_0} \phi$. We may assume that ϕ has the form guaranteed by by Lemma 2, and by Lemma 3, we may assume that the depth of $\mathcal{A}$ is of order $|\phi|^2$. As before, let Φ be the set of subformulas of ϕ. For any interval I and any $\psi \in \Phi$, denote by $S(\psi, I)$ the set of all maximal subformulas χ of ψ such that $\mathcal{A} \models_I \chi$ and the major connective of χ is neither $\wedge$ nor $\vee$. Note that, for any ψ and J with $\mathcal{A} \models_I \psi$, $S(\psi, I)$ entails ψ. We now construct a submodel $\mathcal{A}^*$ of $\mathcal{A}$, starting with the interval I_0 and choosing witnesses, tableau-style, for formulas in Φ. In this construction, (V, E) denotes a tree with nodes V and edges E, Q a subset of V, L a mapping $L : V \rightarrow \mathbb{P}(\Phi)$ and λ a mapping $\lambda : V \rightarrow \mathcal{I}$. We update the values of Q, V, E, L and λ in the course of the construction.

Initialize both Q and V to the singleton $\{v_0\}$ and E to $\emptyset$. Set $L(v_0) = S(\phi, I_0)$ and $\lambda(v_0) = I_0$. Thus, for all $v \in V$, we have $\mathcal{A} \models_{\lambda(v)} L(v)$. This property will be maintained throughout. Now execute the following steps until $Q = \emptyset$:

Select some $v \in Q$ and set $Q := Q \setminus \{v\}$.

For each $\psi \in L(v)$, do the following:

1. If $\psi = \langle e \rangle \pi$, let J be such that $\mathcal{A} \models_{I,J} e$ and $\mathcal{A} \models \pi$. Select $w \notin V$ and set $Q := Q \cup \{w\}$, $V := V \cup \{w\}$, $E := E \cup \{(v, w)\}$, $L(w) := S(\pi, J)$ and $\lambda(w) := J$.
2. If $\psi = \{\alpha\}\pi$, let J be such that $\mathcal{A} \models_{I,J} \alpha$. Select $w \notin V$ and set $Q := Q \cup \{w\}$, $V := V \cup \{w\}$, $E := E \cup \{(v, w)\}$, $L(w) := S(\pi, J)$ and $\lambda(w) := J$.
3. If $\psi = \{\alpha\}_< \pi$, let J be such that $\mathcal{A} \models_{I,J} \alpha$ and let $J' = \text{init}(J, I)$. Select $w, w' \notin V$ and set $Q := Q \cup \{w, w'\}$, $V := V \cup \{w, w'\}$, $E := E \cup \{(v, w), (v, w')\}$, $\lambda(w) := J$, $\lambda(w') := J'$, $L(w) := \emptyset$ and $L(w') := S(\pi, J')$.
4. If O is $\{\alpha\}_> \psi$, proceed symmetrically.
5. If ψ is $\neg\{e\}\top$, and there exist $J \subset I$, $J' \subset I$ with $J \neq J'$ and $J, J' \in \mathcal{A}(e)$, choose any such J, J'. Select $w, w' \notin V$ and set $Q := Q \cup \{w, w'\}$, $V := V \cup \{w, w'\}$, $E := E \cup \{(v, w), (v, w')\}$, $\lambda(w) := J$, $\lambda(w') := J'$, $L(w) := \emptyset$ and $L(w') := \emptyset$.
6. In Steps 1–5, for $u = w$ and $u = w'$, and for every formula $[e']\theta \in \Phi$ such that there exists $L \supset \lambda(u)$ with $\mathcal{A} \models_L [e']\theta$ and $e' \in \mathcal{A}(\lambda(u))$, set $L(u) := L(u) \cup S(\theta, \lambda(u))$.

Steps 1–5 ensure, roughly, that 'existential' modal operators have witnesses; Step 6, by contrast, ensures that 'universal' modal operators are not falsified by these witnesses.

We show that the above construction terminates after finitely many iterations, and that, upon termination, the tree (V, E) satisfies the size bound of the theorem. For consider any path $v_0 \rightarrow \cdots \rightarrow v_m$ through (V, E). Evidently, $\lambda(v_0) \supset \cdots \supset \lambda(v_m)$. From the above construction, for all $i < m$, the total size of $L(v_{i+1})$ must be less than the total size of $L(v_i)$, unless Step 6 adds material at the point where v_{i+1} is added to V. But this requires that $e' \in \mathcal{A}(\lambda(v_{i+1}))$ for at least one event-atom e'. Since the depth of $\mathcal{A}$ is of order $|\phi|^2$, and each application of Step 6 adds at most $|\phi|^2$ symbols to $L(v_{i+1})$, it follows that the length of the path $v_0 \to \cdots \to v_m$ is of order $|\phi|^4$. The bound on the eventual size of V follows from the fact that the out-degree of any node in V is bounded by $2|\phi|$. We note in passing that the steps in the above 'construction' are not required to be effectively computable.

Now let $\mathcal{A}^* = \{\langle J, e\rangle \in \mathcal{A} | \text{for some } v \in V,\ J = \lambda(v)\}$. To establish $\mathcal{A}^* \models_{I_0} \phi$, it suffices to show that, for any interval $v \in V$ and any $\psi \in L(v)$, $\mathcal{A}^* \models_{\lambda(v)} \psi$. We proceed by structural induction on ψ. Denote $\lambda(v)$ by I. The base cases are of the forms $\psi = \top, \bot, \neg\{e\}\top$. The first two of these are trivial. For the case $\psi = \neg\{e\}\top$, if $\psi \in L(v)$, either (i) there is no $J \subset I$ such that $J \in \mathcal{A}(e)$ or (ii) there exist $J \subset I$, $J' \subset I$ with $J \neq J'$ such that $J, J' \in \mathcal{A}(e)$. In the former case, since $\mathcal{A}^* \subseteq \mathcal{A}$, then $\mathcal{A}^* \models_I \psi$. In the latter case, Step 5 ensures that, for some such J, J', we have $w, w' \in V$ with $\lambda(w) = J$ and $\lambda(w') = J'$; hence $J, J' \in \mathcal{A}^*(e)$ and $\mathcal{A}^* \models_I \psi$. The inductive cases are almost as straightforward: the following constitute a representative selection.

1. Suppose ψ is $\langle e\rangle\pi$. Then we have $w \in V$ and and $J \subset I$ such that $\lambda(w) = J$, $S(\pi, J) \subseteq L(w)$, $\langle e, J\rangle \in \mathcal{A}$, and $\mathcal{A} \models_J \pi$. By inductive hypothesis, $\mathcal{A}^* \models_J S(\pi, J)$, and since $\mathcal{A} \models_J \pi$, $S(\pi, J)$ entails π, whence $\mathcal{A}^* \models_J \pi$. By construction, $\langle e, J\rangle \in \mathcal{A}^*$, so that $\mathcal{A}^* \models_I \psi$.

2. Suppose ψ is $[e]\pi$. Let $J \subset I$ with $J \in \mathcal{A}^*(e)$. Since $\mathcal{A}^* \subseteq \mathcal{A}(e)$, we have $J \in \mathcal{A}(e)$; and since by hypothesis $\psi \in L(I)$, we have $\mathcal{A} \models_I \psi$. Hence, $\mathcal{A} \models_J \pi$ and so $S(\pi, J)$ entails π. Consider any $w \in V$ with $\lambda(w) = J$. Step 6 will ensure that $S(\pi, J) \subseteq L(w)$. By inductive hypothesis, $\mathcal{A}^* \models_J S(\pi, J)$, whence $\mathcal{A}^* \models_J \pi$. Hence, $\mathcal{A}^* \models_I \psi$.

3. The remaining cases are handled similarly to Case 1.

□

Corollary 1. *The satisfiability problem for $\mathcal{TPL}$ is in NEXPTIME.*

6 Lower Complexity Bound

Denote by $\mathbb{N}_N$ the natural numbers less than N. Recall that an *exponential tiling problem* is a triple (C, H, V), where $C = \{c_0, \ldots, c_{M-1}\}$ is a set and H and V are binary relations over C. We call the elements of C *colours*, and we call H and V the *horizontal constraints* and the *vertical constraints*, respectively. An *instance* of (C, H, V) is a list $c'_0, \ldots c'_{n-1}$ of elements of C (repetitions allowed). Such an instance is *positive* if there exists a function $\tau : \mathbb{N}_{2^n} \times \mathbb{N}_{2^n} \to C$ such that: (i) $\tau(i, 0) = t'_i$ for all i $(0 \leq i \leq n-1)$; (ii) $\langle \tau(i,j), \tau(i+1,j)\rangle \in H$ for all i, j $(0 \leq i < 2^n - 1, 0 \leq j \leq 2^n - 1)$; (iii) $\langle \tau(i,j), \tau(i,j+1)\rangle \in V$ for all i, j

$(0 \le i \le 2^n - 1, 0 \le j < 2^n - 1)$; and (iv) $\tau(0, 2^n - 1) = c_0$. We refer to τ as a *tiling*. Intuitively, the elements of C represent colours of unit square tiles which must be arranged so as to fill a grid of $2^n \times 2^n$ squares, with the top left-hand square required to have a specific colour. The constraints H (respectively, V) list which colours are allowed to go to the right of (respectively, above) which others. The problem instance $c'_0, \ldots, c'_{n-1}$ lists the colours of the first n tiles in the bottom row.

To show that a problem $\mathcal{P}$ is NEXPTIME-hard, it suffices to show that, for any exponential tiling problem (C, H, V), any instance of (C, H, V) may be encoded, in polynomial time, as an instance of $\mathcal{P}$.

We now proceed to do this where $\mathcal{P}$ is $\mathcal{TPL}$-satisfiability. The main technical challenge is to encode, using a succinct formula of $\mathcal{TPL}$, the information that there are *exactly* 2^{2n} pairwise disjoint intervals satisfying some event-atom t within a given interval I^*. We begin by tackling this problem; the remainder of the reduction is more or less routine.

6.1 Fixing a large number of tiles

In the sequel, we take ψ_0 to be the formula $\{a_0\}\top$ asserting that exactly one event of type a_0 occurs.

Let $m \ge 2$ and let a_0, $a^0_1, \ldots, a^0_{m+1}$, $a^1_1, \ldots, a^1_{m+1}$, $b_1, \ldots, b_m$, $p^0_0, \ldots, p^0_{m-1}$ and $p^1_0, \ldots, p^1_{m-1}$ be event-atoms. To simplify the notation, we write a_0 alternatively as a^0_0 or a^1_0. Let ψ_1 be the conjunction of the following formulas, where $0 \le i < m$, $0 \le h \le 1$ and $0 \le h' \le 1$:

$$\begin{array}{ll} [a^{h'}_i]\{a^h_{i+1}\}\langle p^h_i\rangle\top & [a^{h'}_i]\{a^0_{i+1}\}_{>}\langle a^1_{i+1}\rangle\top \\ [a^{h'}_i]\{b_{i+1}\}\langle p^h_i\rangle\top & [a^{h'}_i]\{p^h_i\}\langle a^{1-h}_{i+2}\rangle\top. \end{array} \tag{1}$$

If $\mathcal{A} \models_{I^*} \psi_1$, then, for all i $(0 \le i < m)$, any subinterval $I \subset I^*$ satisfying either a^0_i or a^1_i includes a unique J satisfying a^0_{i+1} and a unique J' satisfying a^1_{i+1}, with J preceding J'. The interval I also includes a unique L satisfying b_{i+1}; moreover, $L \cap J$ contains an interval K satisfying a^1_{i+2}, and $L \cap J'$ contains an interval K' satisfying a^0_{i+2}. It is best to think of the p^0_i and p^1_i as auxiliary event-atoms by means of which these relationships between J, J', K, K' and L are secured. A representative situation conforming to these constraints is depicted in Fig. 2.

Let $q^0_1, \ldots, q^0_{m-1}$ and $q^1_1, \ldots, q^1_{m-1}$ be event-atoms, and let ψ_2 be the conjunction of the following formulas, where $1 \le i < m$ and $0 \le h \le 1$:

$$\begin{array}{ll} [b_i]\{a^h_{i+1}\}\langle q^{1-h}_i\rangle\top & [b_i]\{a^1_{i+1}\}_{>}\langle a^0_{i+1}\rangle\top \\ [b_i]\{b_{i+1}\}\langle q^h_i\rangle\top & [b_i]\{q^h_i\}\langle a^{1-h}_{i+2}\rangle\top. \end{array} \tag{2}$$

If $\mathcal{A} \models_{I^*} \psi_2$, then, for all i $(1 \le i < m)$, any subinterval $I \subset I^*$ satisfying b_i includes a unique J satisfying a^1_{i+1} and a unique J' satisfying a^0_{i+1}, with J preceding J'. The interval I also includes a unique L satisfying b_{i+1}; moreover, $L \cap J$ contains an interval K satisfying a^1_{i+2}, and $L \cap J'$ contains an interval K' satisfying a^0_{i+2}. It is best to think of the q^0_i and q^1_i as auxiliary event-atoms by

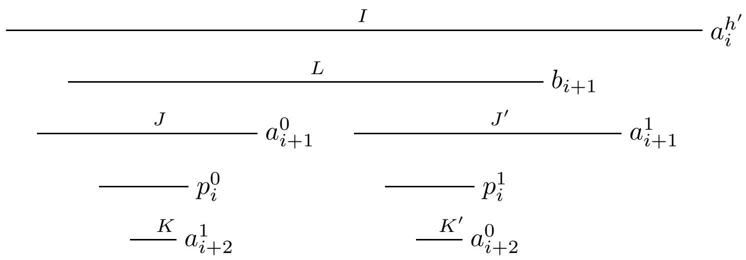


Figure 2: Representative arrangement of intervals under each $a_i^{h'}$-interval.

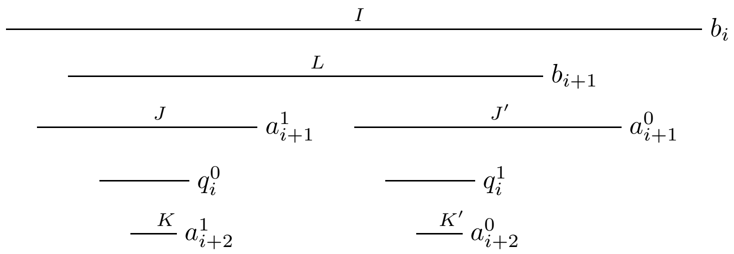


Figure 3: Representative arrangement of intervals under each b_i-interval.

means of which these relationships between J, J', K, K' and L are secured. A representative situation conforming to these constraints is depicted in Fig. 3.

Let $\mathcal{A} \models_{I^*} \psi_0 \wedge \psi_1$. For all i $(0 \leq i \leq m)$, define an *i-witness* inductively as follows:

1. the unique subinterval of I^* satisfying a_0 is a 0-witness;
2. if I is an i-witness, then the unique subinterval of I satisfying a_{i+1}^0 and the unique subinterval of I satisfying a_{i+1}^1 are both $(i+1)$-witnesses;
3. there are no other i-witnesses.

This definition makes sense because each i-witness satisfies either a_i^0 or a_i^1. For each i, the i-witnesses are evidently pairwise disjoint, and alternate on the time-line between those satisfying a_i^0 and those satisfying a_i^1, as depicted in Fig. 4.

Claim 1. *Let $\mathcal{A} \models_{I^*} \psi_0 \wedge \psi_1 \wedge \psi_2$, and let K, K' be consecutive $(i+1)$-witnesses, with $0 \leq i < m$. Then there exists an interval $L \subset I^*$ properly including both K and K', such that L satisfies one of a_i^0, a_i^1 or b_i.*

Proof. We proceed by induction on i. If $i = 0$, the result is immediate.

For the inductive case, suppose the statement of the Lemma holds with $0 \leq i < m-1$; we show the same statement holds with i replaced by $i+1$. Let K, K' be

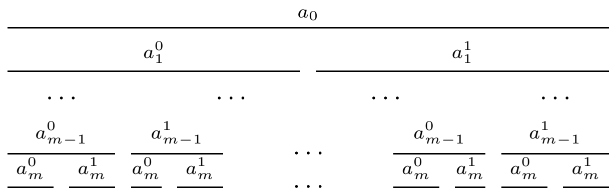


Figure 4: Arrangement of i-witnesses ($0 \leq i \leq m$).

consecutive $(i+2)$-witnesses, then; without loss of generality, we can suppose that K precedes K'. Each $(i+2)$ witness is by definition included in a unique $(i+1)$-witness; so let J be the $(i+1)$-witness such that $K \subset J$ and J' be the $(i+1)$-witness such that $K' \subset J'$. Since K and K' are consecutive, J and J' are identical or consecutive. In the former case, we may put $L = J = J'$, and L satisfies either a_{i+1}^0 or a_{i+1}^1 as required by the Lemma. So assume the latter. By inductive hypothesis, then, J and J' are included within an interval $I \subset I^*$ such that I satisfies a_i^0, a_i^1, or b_i. Moreover, since K and K' are consecutive but not included in a common $(i+1)$-witness, K satisfies a_{i+2}^1 and K' satisfies a_{i+2}^0.

If I satisfies $a_i^{h'}$ ($0 \leq h' \leq 1$), then ψ_1 guarantees that I includes exactly one interval satisfying a_{i+1}^0 and exactly one interval satisfying a_{i+1}^1, with the former preceding the latter; these must be, respectively, J and J', therefore. Again by ψ_1, J includes exactly one interval satisfying a_{i+2}^1 and J' exactly one interval satisfying a_{i+2}^0; these must be, respectively, K and K', therefore. Thus, we have the arrangement of Fig. 2. In particular, ψ_1 guarantees the existence of an interval L satisfying b_{i+1} and including both K and K', as required by the Lemma.

If I satisfies b_i, then ψ_2 guarantees that I includes exactly one interval satisfying a_{i+1}^1 and exactly one interval satisfying a_{i+1}^0, with the former preceding the latter; these must be, respectively, J and J', therefore. Moreover, by ψ_1, J includes exactly one interval satisfying a_{i+2}^1 and J' exactly one interval satisfying a_{i+2}^0; these must be, respectively, K and K', therefore. Thus, we have the arrangement of Fig. 3. In particular, ψ_2 guarantees the existence of an interval L satisfying b_{i+1} and including both K and K', as required by the Lemma. □

Under the conditions of Claim 1, if K and K' are consecutive i-witnesses (in that order), then no subinterval $H \subset I^*$ satisfying either a_i^0 or a_i^1 can begin after K starts and end before K' ends. For if $i > 0$, we have some $L \subset I^*$ satisfying one of a_{i-1}^0, a_{i-1}^1 or b_{i-1}, with $L \supset K$ and $L \supset K'$. Thus, $L \supset H$, which contradicts either ψ_1 or ψ_2.

Let ψ_3 be the conjunction of the following formulas, where i ($1 \leq i \leq m$):

$$\begin{array}{ll} [a_1^0] \cdots [a_{i-1}^0]\{a_i^0\}_<\langle c_i^0\rangle\top & \{c_i^0\}_>(\{a_i^0\}\top \wedge [a_i^1]\bot) \\ [a_1^1] \cdots [a_{i-1}^1]\{a_i^1\}_>\langle c_i^1\rangle\top & \{c_i^1\}_<(\{a_i^1\}\top \wedge [a_i^0]\bot) \end{array} \quad (3)$$

If $\mathcal{A} \models_{I^*} \psi_0 \wedge \cdots \wedge \psi_3$ and $1 \leq i \leq m$, let J be the first-occurring i-witness. Then there exists a unique subinterval $K \subset I^*$ satisfying c_i^0; J precedes K; and J is the only subinterval of I^* preceding K and satisfying either a_i^0 or a_i^1. In particular, no subinterval of I^* satisfying either a_i^0 or a_i^1 can end before the first i-witness ends. Similarly, no subinterval of I^* satisfying either a_i^0 or a_i^1 can start after the last i-witness starts.

For any $n > 0$, we define a sequence of $(2^n)^2 = 2^{2n}$ consecutively numbered intervals as follows. Set $m = 2n+1$, and let $d_1, \ldots d_{m-1}$ be event atoms. (Think of d_i as representing the ith digit in an $(m-1)$-digit binary numeral, where the first digit is the most significant and the $(m-1)$th the least significant.) Let ψ_4 be the conjunction of the following formulas, where i $(1 \leq i < m)$:

$$[a_i^0][a_m^0][d_i]\bot \qquad [a_i^1][a_m^0]\langle d_i\rangle\top \tag{4}$$

Claim 2. *Let $\mathcal{A} \models_{I^*} \psi_0 \wedge \cdots \wedge \psi_4$, and consider the 2^{2n} m-witness which satisfy a_m^0. Let these intervals be numbered in order of temporal precedence as $J_0, \ldots J_{2^{2n}-1}$. In that case, for all k $(0 \leq k < 2^{2n})$, and all i $(1 \leq i \leq 2n)$ the ith digit $k[i]$ in the $2n$-digit binary numeral for k (counting the most significant as the first) is given by:*

$$k[i] = \begin{cases} 1 & \textit{if } \mathcal{A} \models_{J_k} \langle d_i\rangle\top \\ 0 & \textit{otherwise.} \end{cases}$$

Proof. By inspection of Fig 4. □

Finally, let ψ_5 be the conjunction of the following formulas, where $0 \leq h \leq 1$:

$$[a_m^0][a_m^h]\bot. \tag{5}$$

Claim 3. *Let $\mathcal{A} \models_{I^*} \psi_0 \wedge \cdots \wedge \psi_5$. Then there exist exactly 2^{2n} subintervals of I^* satisfying a_m^0.*

Proof. Certainly, there are exactly 2^{2n} m-witnesses which satisfy a_m^0. Suppose $J \subset I^*$ and J satisfies a_m^0, but J is not an m-witness. By ψ_5, J may not properly include any m-witness. Hence, the following possibilities are exhaustive: (i) J ends before the first m-witness ends; (ii) J begins after one m-witness begins and ends before the next one ends; and (iii) J begins after the last m-witness begins. But we have already ruled out all these possibilities. Hence, all subintervals of I^* satisfying a_m^0 are m-witnesses. □

Let us refer to the $(2^n)^2$ intervals identified in Claims 2 and 3 as *tiles*, and let us write a_m^0 more suggestively as t. Say that the kth tile in the usual temporal order $(0 \leq k < 2^{2n})$ *has index* k. If J is any tile, denote its index by k_J. In that case, Claim 2 lets us read $\mathcal{A} \models_J \langle d_i\rangle\top$ as 'saying' that the ith digit in the $2n$-digit binary representation of k_J is 1.

6.2 Organizing the tiles into a grid

By grouping the tiles into 2^n blocks, each containing 2^n consecutive tiles, we have a $2^n \times 2^n$ grid. If J and J' are tiles, then J' lies immediately above J in the grid in case $k_{J'} = k_J + 2^n$; similarly, J' lies immediately to the right of J in the grid in case $k_{J'} = k_J + 1$ and the last n bits of k_J are not all 1s. Let v be an event-atom. We now write formulas ensuring that, for all tiles J, J' such that $k_{J'} = k_J + 2^n$, there exists an interval L satisfying v such that J is the first tile included in L and J' is the last.

The first stage is to ensure that there are enough instances of v. Let $f_0, f_1^0 \ldots f_{2n}^0, f_1^1 \ldots f_{2n}^1$ be event atoms, and again write f^0 alternatively as f_0^0 or f_0^1. Let ψ_6 be the conjunction of the following formulas, where $0 \leq i < 2n$, $1 \leq j \leq n$ and $0 \leq h \leq 1$:

$$\begin{array}{l} \langle f_0 \rangle \\ [f_i^h](\langle f_{i+1}^0 \rangle \top \wedge \langle f_{i+1}^1 \rangle \top) \\ [f_{i+1}^0][v]\{t^f\}[d_{i+1}]\bot \\ [f_{i+1}^1][v]\{t^f\}\langle d_{i+1} \rangle \top \\ [f_j^0][f_{2n}^h]\langle v \rangle \top. \end{array} \tag{6}$$

If $\mathcal{A} \models_{I^*} \psi_0 \wedge \cdots \psi_6$, a little thought shows that every tile $J_0, \ldots, J_{2^{2n}-2^n-1}$ is the first tile included in some interval satisfying v. (Notice in particular how the modal operators $[f_j^0]$, where $1 \leq j \leq n$, ensure that the formulas $[f_j^0][f_{2n}^h]\langle v \rangle \top$ do not imply the existence of such intervals for tiles J_k with $k > 2^{2n} - 2^n - 1$, that is to say, for values of k which have all 1's in their first n-bits.) We then need only ensure that all such intervals contain exactly $2^n + 1$ consecutive tiles. As a preliminary, let let $d_1^*, \ldots d_{2n}^*$ be event-atoms, and ψ_7 be the conjunction of the following formulas, where $1 \leq i \leq n$:

$$[t]\left(\langle d_i^* \rangle \top \leftrightarrow \left([d_i]\bot \wedge \bigwedge_{i<j\leq n} \langle d_j \rangle \top\right)\right) \tag{7}$$

Here and in the sequel, the use of $\neg$ implicit in the connectives $\leftrightarrow$ and $\rightarrow$ is actually completely dispensable, and the Theorem does not hinge on it; we include it because it helps to make the formulas a little more intuitive. The purpose of ψ_7 is to enable us to simulate the incrementation operation on binary numerals. Suppose $\mathcal{A} \models_{I^*} \psi_1 \wedge \cdots \wedge \psi_7$. Then it is routine to check that, for any tile J with k_J in the range $0 \leq k_J \leq 2^{2n} - 2^n - 1$, $\mathcal{A} \models_J \langle d_i^* \rangle \top$ if and only if i is the least integer such that the jth digit in the $2n$-digit binary representation of k_J is 1 for all j in the range $i < j \leq n$. With this interpretation in mind, let

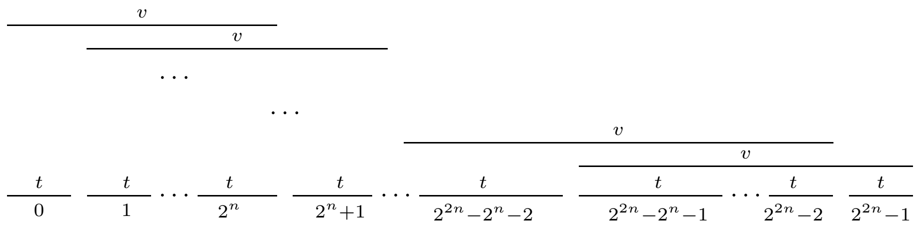


Figure 5: Arrangement of event-atoms indicating vertical neighbourhood in the grid

ψ_8 be the conjunction of the following formulas, where $1 \leq i \leq n$:

$$\begin{aligned}
&\bigwedge_{1\leq i\leq n} [v]\left(\{t\}^f\langle d_i^*\rangle\top \rightarrow \{t\}^l(\bigwedge_{i<j\leq n}[d_j]\bot \wedge \langle d_i\rangle\top)\right)\\
&\bigwedge_{1\leq i\leq n}\bigwedge_{1\leq j<i}[v](\{t\}^f\langle d_i^*\rangle\top \rightarrow (\{t\}^f\langle d_j\rangle\top \leftrightarrow \{t\}^l\langle d_j\rangle\top)) \qquad (8)\\
&\bigwedge_{n<j\leq 2n}[v](\{t\}^f\langle d_j\rangle\top \leftrightarrow \{t\}^l\langle d_j\rangle\top)
\end{aligned}$$

If $\mathcal{A} \models_{I^*} \psi_1 \wedge \cdots \wedge \psi_8$, then we can read ψ_8 as stating that in every subinterval $J \subset I^*$ satisfying v, the indices of the first and last cells included in J differ by precisely 2^n. Pictorially, we have the arrangement of v-satisfying intervals shown in Fig. 5. The corresponding formulas $\psi_9, \ldots, \psi_{12}$ required to establish a suitable arrangement of event-types h encoding horizontal neighbourhood are analogous and need not be spelt out here.

6.3 Encoding Tiling Problems

We are now ready to prove the main result of this section.

Theorem 2. *The satisfiability problem for $\mathcal{TPL}$ is NEXPTIME-hard.*

Proof. Let (C, H, V) be any exponential tiling problem and $c'_0, \ldots c'_{n-1}$ an instance of size n. Setting $m = 2n + 1$, construct the formulas $\psi_1, \ldots, \psi_{12}$ as above. If $C = \{c_0, \ldots, c_{M-1}\}$, take the c_i $(0 \leq i < M)$ to be event-atoms, and let ψ_T be the conjunction of the following two formulas:

$$[t]\bigvee_{0\leq j<M}\langle c_j\rangle\top$$

$$[t]\bigwedge_{0\leq j<j'<M}([c_j]\bot \vee [c_{j'}]\bot).$$

Given a tile J, we regard the realization of an event-atom c_j in a subinterval of J as indicating that the tile J is coloured by c_j. The formula ψ_T simply states that each tile has exactly one colour chosen from C.

Let ψ_H be the conjunction of the following formulas, where $(c_i, c_j) \notin H$:

$$[h](\{t^f\}[c_i]\bot \vee \{t^l\}[c_j]\bot).$$

Similarly, let ψ_V be the conjunction of the following formulas, where $(c_i, c_j) \notin V$:

$$[v](\{t^f\}[c_i]\bot \vee \{t^l\}[c_j]\bot).$$

The motivation for ψ_H and ψ_V should be obvious. Finally, we encode the initial tile c'_0 using the formula

$$\langle t\rangle([d_0]\bot \wedge \cdots \wedge [d_{2n}]\bot \wedge \langle c'_0\rangle\top),$$

and similarly for the $c'_1, \ldots, c'_{n-1}$. Denote the conjunction of all these formulas by ψ_I. From the above constructions, it is routine to verify that the instance $c'_0, \ldots, c'_{n-1}$ of (C, H, V) is positive if and only if

$$\psi_0 \wedge \ldots \wedge \psi_{12} \wedge \psi_T \wedge \psi_H \wedge \psi_V \wedge \psi_I$$

is satisfiable. This completes the reduction. □

7 Conclusion

In this paper, we defined the fragment of temporal English $\mathcal{TPE}$, together with a matching interval temporal logic $\mathcal{TPL}$. The satisfiability problem for $\mathcal{TPL}$ was shown to be complete for the complexity class NEXPTIME. In view of the intimate connection between $\mathcal{TPE}$ and $\mathcal{TPL}$, we take this result to indicate the complexity of performing logical deductions in the fragment of temporal English in question, and thus to give a rough measure of the expressive resources which the grammatical constructions it features—primarily, temporal prepositions and subordinating conjunctions—put at speakers' disposal. By the standards of most interval temporal logics, $\mathcal{TPL}$ has low complexity. In the search for logics of limited expressive power, fragments owing their salience to the syntax of natural language are a natural place to look.

Throughout this paper, we have endeavoured on the one hand to be faithful to the syntax and semantics of temporal constructions in English, and on the other to retain a reasonably perspicuous formal system, amenable to mathematical analysis. These two aims are to some extent antagonistic, of course. Natural languages are products of human biology and human civilization, and as such do not always admit of a comfortable mathematical description. Thus, even the simple fragment of English considered here skirts many delicate issues in syntax, and includes sentences about whose exact semantics even native speakers are uncertain. In this situation, we have occasionally had to legislate, sometimes in whatever way is mathematically most convenient. Nevertheless, while faithfulness to the linguistic data is a virtue, it is all to easy, in pursuit of this virtue, to lose sight of the remarkable logical regularity of the constructions studied here; and it is this regularity that has been to the fore in our investigation. To what extent this analysis can be usefully extended to cover other temporal constructions in English (and other natural languages), and what effects such extensions will have on the complexity of satisfiability in the accompanying logic, remain open.

Appendix: The grammar rules for $\mathcal{TPE}$

Syntax		Open-class lexicon
$\mathrm{S} \rightarrow \mathrm{S}, \mathrm{PP}$	$\mathrm{PP} \rightarrow \mathrm{P}_{\mathrm{N},D}, \mathrm{NP}_D$	$\mathrm{S}^0 \rightarrow$ an interrupt was received/int-rec
$\mathrm{S} \rightarrow \mathrm{S}, \mathrm{Conj}, \mathrm{S}$	$\mathrm{PP} \rightarrow \mathrm{P}_{\mathrm{S},D}, \mathrm{S}^1_D$	$\mathrm{S}^0 \rightarrow$ the main process ran/main
$\mathrm{S} \rightarrow \mathrm{S}^0$	$\mathrm{NP}_D \rightarrow \mathrm{Det}_D, \mathrm{N}^1_D$	...
$\mathrm{S} \rightarrow \mathrm{Neg}, \mathrm{S}^0$	$\mathrm{N}^1_D \rightarrow \mathrm{N}^0$	$\mathrm{N}^0 \rightarrow$ cycle/cyc
$\mathrm{S}^1_D \rightarrow \mathrm{S}^0$	$\mathrm{N}^1_! \rightarrow \mathrm{OAdj}, \mathrm{N}^0$	$\mathrm{N}^0 \rightarrow$ run of the main process/main
$\mathrm{S}^1_! \rightarrow \mathrm{S}^0, \mathrm{OAdv}$	$\mathrm{S}^1_D \rightarrow \mathrm{S}^0$	...
	$\mathrm{S}^1_! \rightarrow \mathrm{S}^0, \mathrm{OAdv}$	

Closed-class lexicon

$\mathrm{Det}_\forall \rightarrow$ every/$[\ ]$	$\mathrm{OAdj} \rightarrow$ first/f	$\mathrm{OAdv} \rightarrow$ for the first time/f
$\mathrm{Det}_\exists \rightarrow$ some/$\langle\ \rangle$	$\mathrm{OAdj} \rightarrow$ last/l	$\mathrm{OAdv} \rightarrow$ for the last time/l
$\mathrm{Det}_! \rightarrow$ the/$\{\ \}$	$\mathrm{P}_{\mathrm{N},D} \rightarrow$ during/$=$	$\mathrm{P}_{\mathrm{S},!} \rightarrow$ when/$(=, \{\ \})$
$\mathrm{Neg} \rightarrow$ not		$\mathrm{P}_{\mathrm{S},\forall} \rightarrow$ whenever/$(=, [\ ])$
$\mathrm{Conj} \rightarrow$ and/$\wedge$	$\mathrm{P}_{\mathrm{N},!} \rightarrow$ until/$<$	$\mathrm{P}_{\mathrm{S},!} \rightarrow$ until/$(<, \{\ \})$
$\mathrm{Conj} \rightarrow$ or/$\vee$	$\mathrm{P}_{\mathrm{N},!} \rightarrow$ before/$<$	$\mathrm{P}_{\mathrm{S},!} \rightarrow$ before/$(<, \{\ \})$
	$\mathrm{P}_{\mathrm{N},!} \rightarrow$ after/$>$	$\mathrm{P}_{\mathrm{S},!} \rightarrow$ after/$(>, \{\ \})$.

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