Fuzzy Reasoning over Sparse Rule Bases via Scale and Move Transformation Based Interpolation

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Abstract: Classic fuzzy inferences such as Mamdani can only handle conventional *dense* fuzzy rule bases, but not *sparse* ones. Fuzzy interpolative reasoning is thus developed to deal with this. A novel interpolation method via scale and move transformations has been proposed and the experimental work shows its potential as compared with the classic inference approaches.

Introduction

- Conventional fuzzy reasoning can only work on a dense rule base (Fig. 1 left) to infer a result, but not on a sparse one (Fig. 1 right).

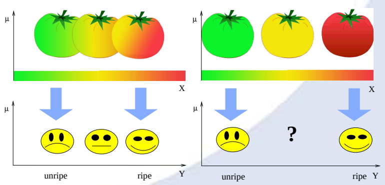

Figure 1. Fuzzy reasoning

- Fuzzy interpolative reasoning supports inference in sparse rule bases and helps their simplification.
- Basic concept of interpolation: Given two fuzzy rules $A_1 \to B_1$ and $A_2 \to B_2$, and an observation A^* between A_1 and A_2, derive B^* by interpolation.
- Issues with existing fuzzy interpolation approaches: 1) results may be non-convex; 2) interpolation may be hard to implement in real-life applications; and 3) lack of flexibility.

Proposed Method

1. Define Representative Values (RVs)

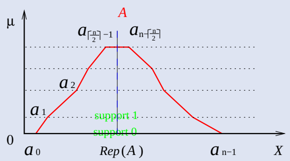

Figure 2. A general fuzzy set

- The RV captures the fuzzy set's overall location.
- The simplest RV definition is the average of all points: $Rep(A) = \frac{1}{n}\sum_{i=0}^{n-1} a_i$.
- Alternative RV definitions such as *weighted average*, *core center* [1] may be adopted, which provides a degree of freedom for various requirements.

2. Construct the Intermediate Rule

Given $A_1 \to B_1$, $A_2 \to B_2$ and observation A^*, construct a new fuzzy rule $A' \to B'$ with $Rep(A') = Rep(A^*)$.

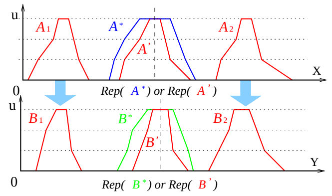

Figure 3. Construct the intermediate rule

3. Introduce Similarity Reasoning

- The more similar A^* to A', the more similar B^* to B'. Similarity is measured by the proposed scale and move transformations.
- **Scale Transformation (Fig. 4 left)**
  - Given *scale rates* s_i, $i = \{0, \ldots, \lfloor\frac{n}{2}\rfloor - 1\}$, transform the fuzzy set with support $(a_{n-1-i} - a_i)$ to a new fuzzy set with support $(s_i * (a_{n-1-i} - a_i))$, while keeping $Rep(A') = Rep(A)$ and $\frac{a'_{i+1}-a'_i}{a'_{n-1-i}-a'_{n-2-i}} = \frac{a_{i+1}-a_i}{a_{n-1-i}-a_{n-2-i}}$.
  - Scale ratio $\mathbb{S}_i$ is introduced to represent the portion of the maximal possible scale (in order not to cause non-convexity) of support i.
  - Given two fuzzy sets, the scale rates can be calculated. The closer the scale rates to 1, the more similar (in terms of the support lengths) the two fuzzy sets.

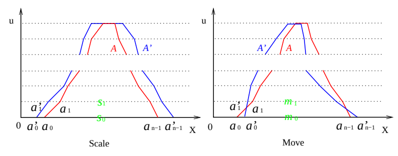

Figure 4. Scale and move transformations

- **Move Transformation (Fig. 4 right)**
  - Given moving distance l_i $(i = \{0, \ldots, \lceil\frac{n}{2}\rceil - 2\})$, transform the fuzzy set from the starting location a_i to a new starting position $a'_i = a_i + l_i$ while keeping $Rep(A') = Rep(A)$ and the same lengths of supports (i.e. $a'_{n-1-i} - a'_i = a_{n-1-i} - a_i$).
  - Move ratio $\mathbb{M}_i$ is introduced to represent the portion of the maximal possible move (in order not to cause non-convexity) of support i.
  - Given two fuzzy sets, the move ratios can be calculated. The closer the move ratio to 0, the more similar (in terms of the position) the two fuzzy sets.
- The scale and move transformations are extended to interpolate multiple fuzzy rules, with each rule having multiple variables associated with arbitrarily complex fuzzy sets.

4. Method Summary

1. Calculate scale ratios of supports from A' to A^*.
2. Apply scale transformation to A' and B' using scale ratios as calculated in step 1 to obtain A'' and B'', respectively.
3. Calculate the move ratios from A'' to A^*.
4. Apply move transformation to B'' using the move ratios as calculated in step 3 to obtain B^*, which is the interpolation result.

A Realistic Application

- Use computer activity data base which has 8192 instances, with each having 22 numeric attributes.
- Employ Fuzzy ID3 to generate the fuzzy rules. Fig. 5 shows the relative squared error (relative to the simple average predictor) with respect to the number of fuzzy partitions and minimal leaf objects (used to terminate training).

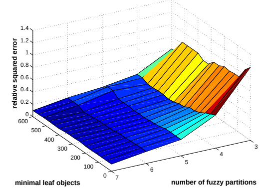

Figure 5. Fuzzy ID3 results

- Consider the trade-off of the fuzzy rule number and the prediction accuracy, an optimal resultant rule base (with 47 rules) is chosen for comparison.
- Mamdani results in an error rate of 13.29% (with some data uncovered by this inference).
- The proposed interpolation reasoning covers all test data, leading to lower error rates with different RV definitions.

RV type	average	w_average	core center
error rate	6.92%	6.28%	7.20%

References

[1] Z. H. Huang and Q. Shen, "Fuzzy interpolation reasoning via scale and move transformations," to appear on *IEEE Trans. on Fuzzy Systems*, 2004.

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